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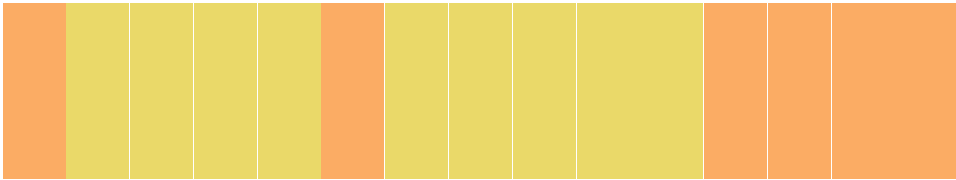
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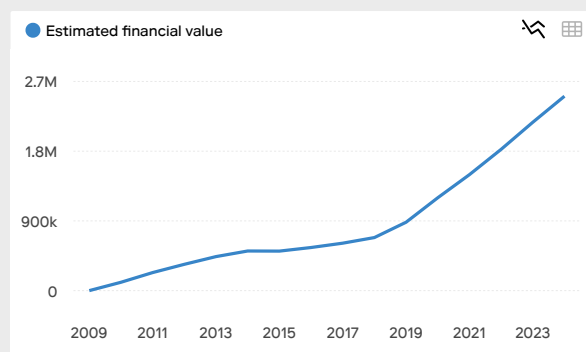
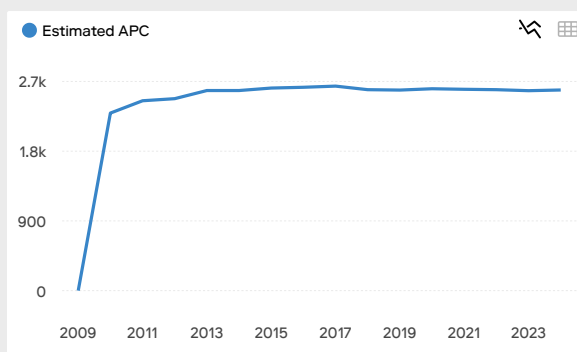
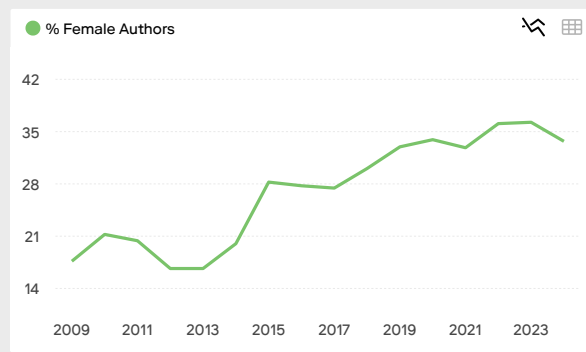
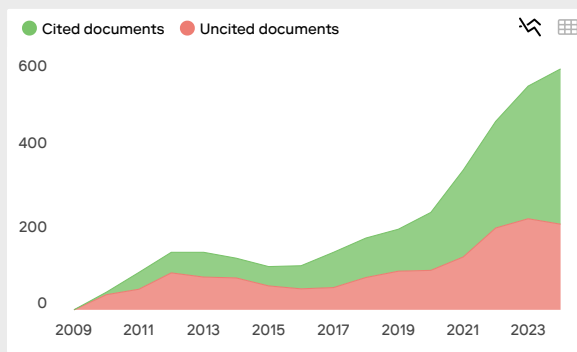
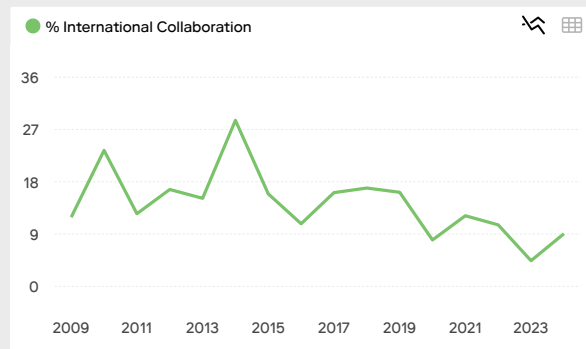
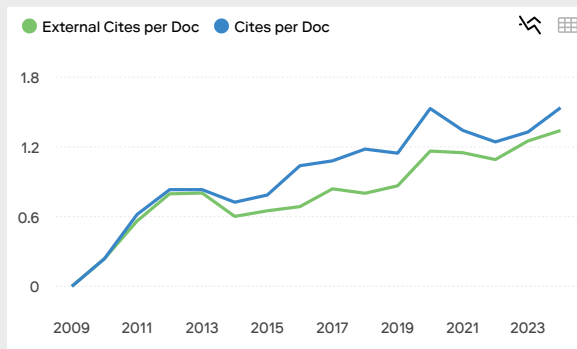
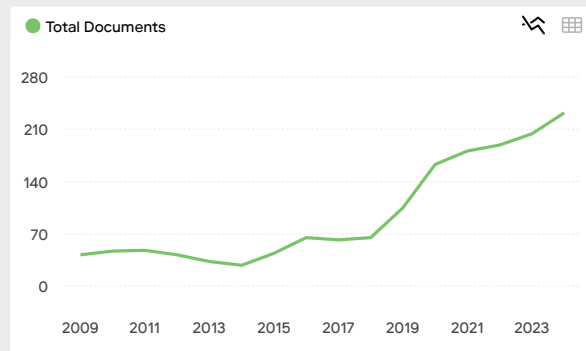
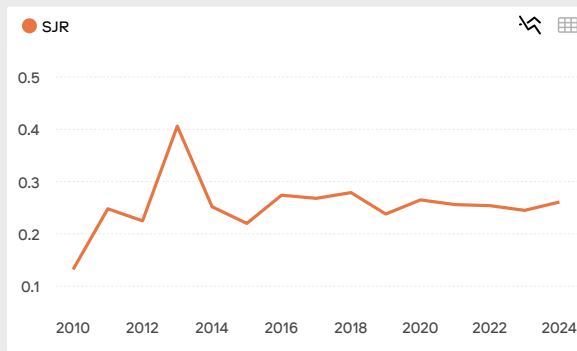
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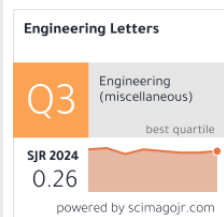
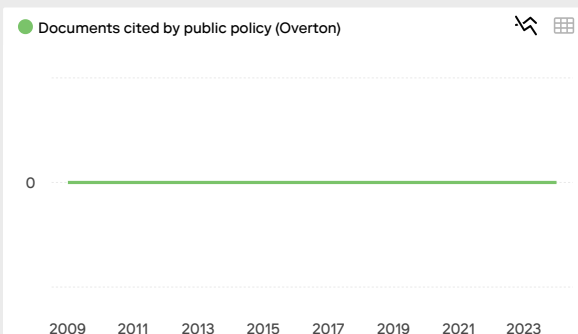
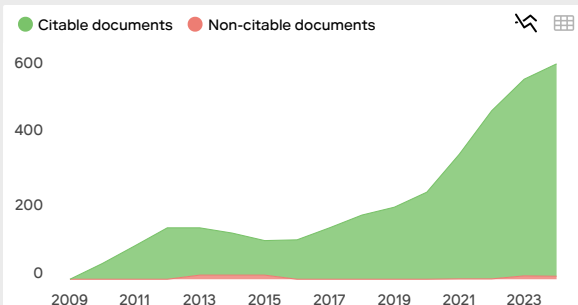
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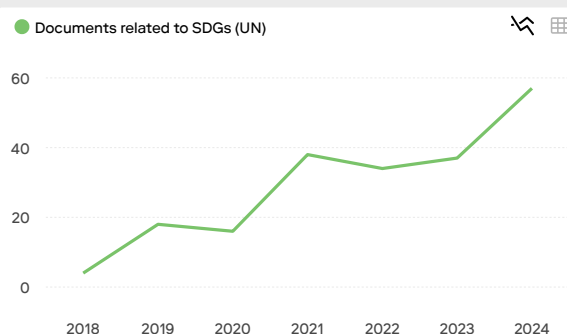
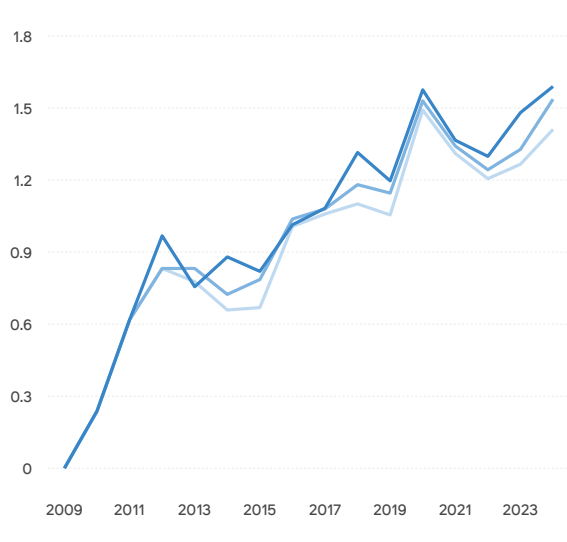




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Performance Enhancement of c-chart via Bayesian Method

Yadpirun Supharakonsakun

Abstract—This study proposes a Bayesian framework for constructing c-charts to monitor Poisson-distributed processes, incorporating both squared error and entropy loss functions to estimate the Poisson parameter (λ) and derive control limits. The performance of the proposed methods is evaluated using Average Run Length (ARL), Standard Deviation of Run Lengths (SDRL), Average Extra Quadratic Loss (AEQL), and Performance Comparison Index (PCI). Simulation results show that the Bayesian c-charts, particularly under the entropy loss function, outperform the classical method by offering earlier and more consistent detection of process shifts, especially for small to moderate inspection unit sizes. The entropy-based estimator, newly introduced in this study, yields the lowest ARL_1 and $SDRL_1$ values, with PCI values close to 1, highlighting its superior sensitivity to process shifts. To further validate practical applicability, the proposed approach was applied to a real industrial dataset involving defect detection in glass bottles. The Bayesian methods successfully identified process shifts earlier than the classical c-chart, demonstrating their effectiveness in real-world quality control scenarios. These findings suggest that the proposed Bayesian framework provides a viable and efficient alternative to classical methods, particularly in applications requiring timely detection of subtle process deviations.

Index Terms—Bayesian estimation, c-chart, control chart, Poisson distribution, entropy loss function, squared error loss function, Average Run Length (ARL), quality control, performance evaluation, process monitoring

I. INTRODUCTION

MAINTAINING consistent product quality is a fundamental objective in modern manufacturing and service industries. Among various quality control tools, control charts have proven to be effective for monitoring processes and detecting abnormal variations. Particularly, the c-chart is widely used to track the number of nonconformities or defects per inspection unit in situations where defects occur according to a Poisson distribution [1].

Although the traditional c-chart is straightforward and widely adopted due to its simplicity, it has notable limitations. Specifically, it assumes that the defect counts strictly follow a Poisson distribution with a known and stable mean (λ). In practice, this assumption may be violated, leading to reduced performance—particularly when detecting small or gradual process shifts. Such limitations

have prompted researchers to investigate more advanced techniques aimed at improving the sensitivity, adaptability, and overall robustness of control charts [2].

Bayesian methods have received increasing attention in statistical process control (SPC) due to their ability to incorporate prior information and update estimates as new data become available. While these approaches have shown notable improvements in control charts for continuous data, such as the Shewhart and EWMA charts [3], particularly under the assumption of normality [4–5], relatively few studies have focused on attribute control charts like the c-chart.

Several significant Bayesian extensions have been proposed for Poisson-based control charts. For instance, Assareh et al. [6] introduced a Bayesian hierarchical model for estimating change points in Poisson processes, which was applied alongside c-chart, EWMA, and CUSUM-type control charts. Mitchell et al. [7] developed a Bayesian EWMA chart under various loss functions, including squared error, LINEX, and entropy, for processes governed by Poisson distributions. Additional studies have proposed Bayesian c-charts using priors such as Jeffreys' prior [8] and the gamma prior [9], reporting improved ARL and false alarm rates compared to classical (frequentist) methods [10]. More recently, an empirical Bayes approach employing an exponential prior was introduced to further enhance the performance of the c-chart [11]. These studies collectively support the effectiveness of Bayesian methods in improving the utility of SPC.

Further investigations into Bayesian estimation of the Poisson parameter have highlighted the benefits of employing diverse loss functions, particularly squared error and entropy. These methods have demonstrated lower mean squared error (MSE) compared to maximum likelihood estimation (MLE) when estimating small parameter values [13–15]. A variety of loss functions have been utilized in Bayesian inference for different distributions, including exponential distributions under entropy and LINEX loss [16], Laplace distributions under entropy loss [17], gamma distributions [18], and exponentiated exponential distributions [19]. The findings consistently show that Bayesian estimators provide more accurate parameter estimates when the true value is small.

Building on this foundation, our study focuses on enhancing the traditional c-chart through Bayesian estimation of the Poisson parameter (λ). We derive the posterior Bayes estimator of λ under both the squared error and entropy loss functions, then use the Bayesian estimate to define control limits.

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The evaluation of the proposed Bayesian c-chart is using key performance metrics: out-of-control Average Run Length (ARL₁), Average Expected Quality Loss (AEQL), and the Performance Comparison Index (PCI). By integrating parameter uncertainty into the chart design, our method aims to improve detection sensitivity and reliability, particularly in practical settings with fluctuating defect rates or available prior information.

This work not only contributes a novel Bayesian framework for attribute control charts but also offers practical value for quality practitioners seeking to enhance process surveillance strategies.

II. CHARACTERISTIC OF THE C-CHART

The c-chart is a fundamental tool in statistical process control (SPC) used to monitor the number of nonconformities in fixed-size inspection units. Widely implemented across manufacturing and service industries, this chart distinguishes between common-cause variation (natural process fluctuations) and special-cause variation (external influences). By establishing appropriate control limits, it facilitates timely detection of process shifts and supports corrective actions to maintain product quality and process stability.

To construct a c-chart, the average number of nonconformities ($\bar{\lambda}$) is first calculated from the observed data. This average is then used to compute the control limits as follows:

$$\begin{aligned} UCL &= \bar{\lambda} + 3\sqrt{\bar{\lambda}}, \\ CL &= \bar{\lambda} \\ LCL &= \bar{\lambda} - 3\sqrt{\bar{\lambda}}. \end{aligned} \quad (1)$$

In classical statistics, the parameter $\bar{\lambda}$, representing the mean number of nonconformities, is central to c-chart construction. In this study, the estimation of λ is extended into a Bayesian framework, where both symmetric and asymmetric loss functions are utilized. This approach allows a more nuanced evaluation of estimator performance under varying decision-making contexts.

III. BAYESIAN FRAMEWORK FOR ESTIMATING THE POISSON PARAMETER

The c-chart, also known as a count chart, is specifically designed for processes involving discrete, countable events. It is ideal for applications where the number of defects per unit, batch, or defined area must be monitored over time. The chart assumes that the count of nonconformities follows a Poisson distribution, suitable for modeling infrequent events occurring independently over a fixed interval.

The construction of a c-chart relies on the assumption that the number of nonconformities follows a Poisson distribution, an appropriate model for rare, independent events occurring within a fixed interval of time or space. When inspection units are observed at regular time intervals, the number of nonconformities detected during the i^{th} inspection is assumed to follow a Poisson distribution with a mean parameter λ , as expressed by:

$$f(x_i | \lambda) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}, \quad x_i = 0, 1, 2, \dots; \quad i = 1, \dots, m; \quad \lambda > 0. \quad (2)$$

To implement the Bayesian approach, a conjugate prior distribution is assumed for λ , specifically the Gamma distribution with shape parameter η and rate parameter κ , denoted as $\lambda \sim \text{Gamma}(\eta, \kappa)$, following the guidelines of Supharakonsakun [13] and Song & Kim [20]:

$$\pi(\lambda | \eta, \kappa) = \frac{\kappa^\eta \lambda^{\eta-1} e^{-\kappa\lambda}}{\Gamma(\eta)}, \quad \eta, \kappa > 0. \quad (3)$$

Applying Bayes' theorem, the posterior distribution of λ becomes:

$$h(\lambda | X) \propto L(x | \lambda) \pi(\lambda). \quad (4)$$

Substituting the likelihood and prior leads to the updated posterior distribution:

$$h(\lambda | X) \propto \lambda^{(\sum x_i + \eta - 1)} e^{-(m + \kappa)\lambda}, \quad (5)$$

which is a Gamma distribution with updated parameters:

$$\lambda | X \sim \text{Gamma}\left(\sum_{i=1}^m x_i + \eta, m + \kappa\right). \quad (6)$$

In this study, the mean parameter of the Poisson distribution is estimated using a natural conjugate prior, specifically a Gamma distribution, within the framework of two distinct loss functions: the squared error loss and the entropy loss.

A. The Squared Error Loss Function

The Bayesian estimator for the Poisson parameter λ under the squared error loss function is defined as:

$$L(\lambda, \hat{\lambda}) = (\lambda - \hat{\lambda})^2. \quad (7)$$

As noted by Supharakonsakun [13], the Bayesian estimator that minimizes the squared error loss—denoted by $\hat{\lambda}_{SELF}$, corresponds to the posterior mean of the distribution. Specifically, the point estimate of λ under this loss function is given by:

$$\hat{\lambda}_{SELF} = E[\lambda | X]. \quad (8)$$

Using the posterior distribution derived from a Gamma prior, the estimator takes the following closed-form expression:

$$\hat{\lambda}_{SELF} = \frac{\sum_{i=1}^m x_i + \eta}{m + \kappa}. \quad (9)$$

B. The Entropy Loss Function

In scenarios where the consequences of underestimation and overestimation are not symmetric, an asymmetric loss function such as the entropy loss may be more appropriate. Originally introduced by Dey and Liu [23], and later recommended by Calabria and Pulcini [21] and Yahgmaei et al. [22], the entropy loss function is defined as:

$$L(\lambda, \hat{\lambda}) = \frac{\hat{\lambda}}{\lambda} - \log\left(\frac{\hat{\lambda}}{\lambda}\right) - 1. \quad (10)$$

The Bayesian estimator under the entropy loss function, denoted as $\hat{\lambda}_{ENLF}$, is obtained from the posterior expectation of the reciprocal of λ and is expressed as [14]:

$$\hat{\lambda}_{ENLF} = \left(E \left[\frac{1}{\lambda} | X \right] \right)^{-1}. \quad (11)$$

Given the posterior distribution of parameter λ , the posterior expectation $E[\lambda^{-1} | X]$ is calculated as:

$$E[\lambda^{-1} | X] = \frac{m + \kappa}{\sum_{i=1}^m x_i + \eta - 1}, \quad (12)$$

which leads to the final form of the entropy-based Bayesian estimator:

$$\hat{\lambda}_{ENLF} = \frac{\sum_{i=1}^m x_i + \eta - 1}{m + \kappa}. \quad (13)$$

These two estimators—under squared error and entropy loss—will be used to construct Bayesian c-charts. Their performance will be systematically evaluated using simulation methods across various settings of inspection unit sizes m and prior hyperparameters (η, κ) , as detailed in the following section.

IV. SIMULATION DESIGN AND PERFORMANCE EVALUATION

This study evaluates the performance of classical and Bayesian control charts using multiple metrics: Average Run Length (ARL), Standard Deviation of Run Length (SDRL), Average Extra Quadratic Loss (AEQL), and the Performance Comparison Index (PCI). The Bayesian approach is implemented under both squared error (symmetric) and entropy (asymmetric) loss functions using informative prior distributions.

Control limits are constructed based on posterior estimates of λ , with simulations conducted across different hyperparameter settings, $(\eta, \kappa) = (3, 2), (4, 0.5), (4, 1), (5, 1), (1, 1), (1, 2), (2, 2)$ and $(2, 1)$, and various inspection unit size $m = 10, 30$, and 50 . Each configuration is evaluated through 100,000 Monte Carlo simulations. For benchmarking, results are compared with the Chakraborti and Human method.

The in-control ARL (ARL_0), representing the average number of samples before a false alarm, is fixed at 370. The corresponding SDRL is given by:

$$SDRL_0 = \sqrt{\frac{1 - \alpha}{\alpha^2}}, \quad (14)$$

where α is the Type I error rate. For out-of-control conditions, the $SDRL_1$ is evaluated as:

$$SDRL_1 = \sqrt{\frac{1 - \beta}{\beta^2}}, \quad (15)$$

where β denotes the Type II error rate. Lower values of ARL_1 and $SDRL_1$ are preferred, as they indicate quicker and more consistent detection of process changes.

To further evaluate performance, the Average Extra Quadratic Loss (AEQL), which accounts for the shift size δ , is computed as follows [24]:

$$AEQL = \frac{1}{\Delta} \sum_{\delta_i = \delta_{\min}}^{\delta_{\max}} \delta_i \times ARL_{\delta_i}. \quad (15)$$

Lower AEQL values signify more accurate and cost-effective performance during process shifts.

Lastly, the Performance Comparison Index (PCI) is

defined as the ratio of the AEQL of a given chart to the minimum AEQL among all methods evaluated:

$$PCI = \frac{AEQL_{\text{method}}}{AEQL_{\text{min}}}. \quad (16)$$

A PCI value of 1 indicates the most efficient method. Charts with lower PCI are considered superior in terms of balancing sensitivity to shifts with false alarm rates.

TABLE I
CONTROL LIMIT WIDTHS OF THE C-CHART

m	η, κ	Classical	BAYESSELF	BAYESENLF
10	3,2	2.98605000	3.59080000	3.63200000
10	4,0.5	2.98605100	3.02405000	3.03605000
10	4,1	2.98606000	3.14940000	3.17800000
10	5,1	2.98610300	3.11908000	3.14913233
10	1,1	2.98605060	3.25650000	3.29374682
10	1,2	2.98620000	3.67400000	3.71600000
10	2,2	2.98612000	3.63200000	3.67500000
10	2,1	2.98601000	3.21620000	3.25000000
30	3,2	3.03200000	3.27800000	3.28600000
30	4,0.5	3.03370000	3.04400000	3.05485000
30	4,1	3.03400000	3.11000000	3.12300000
30	5,1	3.03140000	3.10000000	3.11200000
30	1,1	3.03000000	3.14740000	3.16560000
30	1,2	3.03000000	3.30000000	3.31600000
30	2,2	3.03180000	3.28425000	3.30020000
30	2,1	3.03210000	3.13600000	3.15100000
50	3,2	3.05000000	3.20800000	3.21700000
50	4,0.5	3.05000000	3.06200000	3.07100000
50	4,1	3.05000000	3.10650000	3.11500000
50	5,1	3.05000000	3.10000000	3.11000000
50	1,1	3.05000000	3.13000000	3.14000000
50	1,2	3.05000000	3.22600000	3.23500000
50	2,2	3.05000000	3.21540000	3.22400000
50	2,1	3.05000000	3.12500000	3.13400000

The simulation study was conducted using the R program to evaluate the performance of three methods by considering various shift sizes: 0.1, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0, 2.5, 3.0, 5.0, 10.0, and 20.0. With the in-control average run length (ARL_0) set to 370, the control limit width was appropriately determined and is reported in Table 1.

Table 2 presents the ARL and SDRL values obtained from both classical and Bayesian methods for inspection unit sizes 10, 30, and 50, with hyperparameters $(\eta, \kappa) = (3, 2)$. The Bayesian method under the entropy loss function consistently provides lower ARL_1 and $SDRL_1$ values across all shift magnitudes and inspection unit sizes. Furthermore, the AEQL and PCI values in Table 2 confirm the superior performance of the Bayesian method under the entropy loss function, yielding the lowest AEQL and a PCI of 1, which reinforces its effectiveness in detecting process shifts.

Table 3 shows the ARL and SDRL values for the same methods and inspection unit sizes but with hyperparameters $(\eta, \kappa) = (4, 0.5)$. For $m = 10$, the Bayesian method under the squared error loss function produces lower ARL_1 and $SDRL_1$ across all shift sizes. For shift sizes ≥ 0.1 , the entropy loss-based Bayesian method performs better. For $m = 30$, the squared error loss function performs better for shift sizes < 1.5 , whereas the entropy loss function performs equally or better for shift sizes ≥ 1.5 . For $m = 50$, the classical method demonstrates superior performance with

lower ARL_1 and $SDRL_1$ across all shift sizes. AEQL and PCI values in Table 3 further confirm the efficiency of the entropy-based Bayesian method for $m = 10$ and 30, and the classical method's superior performance for $m = 50$.

Table 4 presents ARL and SDRL values with hyperparameters $(\eta, \kappa) = (4, 1)$. For $m = 10$, the Bayesian method under the entropy loss function provides the lowest ARL_1 and $SDRL_1$ values. For $m = 30$, the squared error loss function performs better for shift sizes < 10.0 , while the entropy loss function matches or exceeds its performance for shift sizes ≥ 10.0 . For $m = 50$, the entropy loss-based Bayesian method again proves superior, showing the lowest ARL_1 and $SDRL_1$ across all shift sizes. The AEQL and PCI

values reported in Table 4 further substantiate the superiority of the Bayesian method under the entropy loss function. Specifically, for $m = 10$ and 30, this method achieves the lowest AEQL together with a PCI of 1, which clearly demonstrates its effectiveness and reliability in detecting process shifts. When m increases to 50, however, the classical method exhibits relatively good performance in identifying mean shifts, indicating that while the Bayesian approach under entropy loss generally provides consistent advantages, the classical approach may still offer competitive results in scenarios involving larger sample sizes.

TABLE II
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 3, \kappa = 2$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	368.0835	367.5831	372.2240	371.7237	370.2360	369.7357
	0.1	341.3443	340.8440	335.3374	334.8370	333.4911	332.9908
	0.3	295.0009	294.5005	273.0315	272.5310	271.5756	271.0752
	0.5	251.2264	250.7259	220.2527	219.7521	219.0253	218.5247
	0.7	211.4375	210.9369	178.4422	177.9415	177.4023	176.9016
	1.0	164.2734	163.7727	133.7634	133.2625	132.9988	132.4978
	1.5	107.6290	107.1278	83.2874	82.7859	82.8053	82.3037
	2.0	70.5734	70.0716	54.1180	53.6157	53.7942	53.2919
	2.5	47.8460	47.3433	36.8349	36.3315	36.6146	36.1112
	3.0	32.9933	32.4895	25.7696	25.2647	25.6132	25.1082
	5.0	9.8618	9.3485	8.2467	7.7306	8.1994	7.6831
	10.0	1.9973	1.4114	1.8632	1.2682	1.8574	1.2619
	20.0	1.0273	0.1675	1.0223	0.1511	1.0222	0.1505
AEQL		179.8683		151.6635		150.9915	
PCI		1.1912		1.0045		1.0000	
30	0.0	370.7023	370.2020	372.1693	371.6690	371.4236	370.9232
	0.1	335.5168	335.0164	335.5797	335.0793	334.8953	334.3950
	0.3	273.4316	272.9312	270.5812	270.0807	270.0184	269.5179
	0.5	224.3158	223.8153	220.9769	220.4763	220.5221	220.0216
	0.7	184.1313	183.6306	180.2696	179.7689	179.9506	179.4499
	1.0	138.0050	137.5040	134.1754	133.6745	133.9399	133.4390
	1.5	88.1376	87.6362	85.6528	85.1513	85.4920	84.9905
	2.0	58.0843	57.5821	56.4586	55.9563	56.3762	55.8740
	2.5	39.6245	39.1213	38.5808	38.0776	38.5111	38.0078
	3.0	27.7568	27.2522	27.0872	26.5825	27.0516	26.5469
	5.0	8.9514	8.4366	8.7961	8.2810	8.7870	8.2719
	10.0	1.9597	1.3714	1.9465	1.3574	1.9455	1.3563
	20.0	1.0254	0.1612	1.0248	0.1595	1.0248	0.1594
AEQL		159.2520		156.4168		156.2357	
PCI		1.0193		1.0012		1.0000	
50	0.0	370.2947	369.7944	369.1104	368.6101	369.0563	368.5560
	0.1	334.4773	333.9769	333.1263	332.6259	333.0820	332.5816
	0.3	270.0728	269.5723	268.9225	268.4221	268.8839	268.3835
	0.5	220.4730	219.9724	219.3516	218.8510	219.3205	218.8199
	0.7	180.2672	179.7665	179.2335	178.7328	179.2086	178.7079
	1.0	135.1892	134.6883	134.2746	133.7737	134.2588	133.7579
	1.5	85.7690	85.2676	85.2152	84.7137	85.2061	84.7046
	2.0	56.8680	56.3657	56.5102	56.0079	56.5009	55.9987
	2.5	38.9450	38.4417	38.7245	38.2213	38.7185	38.2152
	3.0	27.4483	26.9437	27.2974	26.7927	27.2937	26.7890
	5.0	8.9340	8.4191	8.8995	8.3846	8.8984	8.3835
	10.0	1.9650	1.3771	1.9629	1.3748	1.9628	1.3747
	20.0	1.0255	0.1616	1.0253	0.1612	1.0253	0.1612
AEQL		157.4783		156.8361		156.8200	
PCI		1.0042		1.0001		1.0000	

TABLE III
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 4, \kappa = 0.5$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	375.2663	374.7659	372.9671	372.4668	377.3031	376.8028
	0.1	348.1869	347.6865	343.1907	342.6903	345.3385	344.8381
	0.3	299.6878	299.1874	291.9387	291.4383	291.3925	290.8921
	0.5	254.9245	254.4240	246.1235	245.6230	243.8571	243.3565
	0.7	215.7930	215.2925	206.6000	206.0994	203.9031	203.4025
	1.0	166.5486	166.0478	158.0772	157.5764	155.0540	154.5532
	1.5	109.0859	108.5847	102.3976	101.8964	99.9667	99.4655
	2.0	70.9377	70.4359	66.0555	65.5536	64.5205	64.0185
	2.5	48.3722	47.8696	45.0735	44.5707	43.9648	43.4619
	3.0	33.3617	32.8579	31.1483	30.6442	30.4212	29.9170
	5.0	19.6426	19.1360	18.1431	17.6360	17.5557	17.0483
	10.0	2.7136	2.1564	2.6211	2.0613	2.5856	2.0248
	20.0	1.0537	0.2378	1.0503	0.2298	1.0490	0.2267
AEQL		206.3191		195.6163		191.8641	
PCI		1.0753		1.0196		1.0000	
30	0.0	372.6642	372.1639	372.7707	372.2703	375.6366	375.1362
	0.1	335.7889	335.2885	335.0705	334.5701	337.1958	336.6954
	0.3	273.6532	273.1527	272.8057	272.3053	274.1679	273.6675
	0.5	224.1883	223.6877	222.9963	222.4958	223.7253	223.2247
	0.7	183.6042	183.1035	182.2666	181.7660	182.6285	182.1278
	1.0	137.9429	137.4420	136.8332	136.3323	136.9543	136.4534
	1.5	87.8583	87.3569	87.1063	86.6049	87.0843	86.5829
	2.0	58.0437	57.5415	57.5815	57.0793	57.5474	57.0452
	2.5	39.0960	39.0468	39.2406	38.7373	39.2127	38.7094
	3.0	27.8201	27.3155	27.6217	27.1171	27.6006	27.0960
	5.0	8.9658	8.4510	8.9240	8.4092	8.9179	8.4030
	10.0	1.9606	1.3724	1.9572	1.3688	1.9563	1.3678
	20.0	1.0254	0.1614	1.0253	0.1609	1.0252	0.1608
AEQL		158.9918		158.3758		158.3622	
PCI		1.0040		1.0001		1.0000	
50	0.0	371.5520	371.0517	374.2404	373.7401	374.5352	374.0348
	0.1	333.8154	333.3150	336.2583	335.7579	336.5408	336.0404
	0.3	270.4021	269.9016	272.1334	271.6329	272.2756	271.7752
	0.5	220.7488	220.2483	222.0084	221.5078	222.0990	221.5984
	0.7	180.4023	179.9016	181.3732	180.8725	181.4161	180.9154
	1.0	135.6308	135.1299	136.2836	135.7827	136.3031	135.8022
	1.5	86.1177	85.6162	86.4978	85.9963	86.5037	86.0023
	2.0	56.9837	56.4815	57.2195	56.7173	57.2209	56.7187
	2.5	38.8428	38.3396	38.9945	38.4913	38.9948	38.4916
	3.0	27.4903	26.9857	27.5950	27.0904	27.5951	27.0905
	5.0	8.9424	8.4276	8.9722	8.4574	8.9722	8.4574
	10.0	1.9670	1.3791	1.9702	1.3826	1.9702	1.3826
	20.0	1.0255	0.1617	1.0256	0.1620	1.0256	0.1620
AEQL		157.6324		158.1266		158.1343	
PCI		1.0000		1.0031		1.0032	

Table 5 presents the ARL and SDRL values obtained from both classical and Bayesian methods for inspection unit sizes $m = 10, 30$, and 50 , with the hyperparameters $(\eta, \kappa) = (5, 1)$. When $m = 10$, the Bayesian method under the entropy loss function consistently provides lower ARL_1 and $SDRL_1$ values across all shift magnitudes. For $m = 30$, the Bayesian method under the squared error loss function performs better for small shift sizes (less than 1.00), while the entropy loss function offers superior performance for shifts of 1.00 and above. At $m = 50$, the classical method demonstrates the highest efficiency, yielding the lowest ARL_1 and $SDRL_1$ values across all shift sizes. Table 5 shows

that the Bayesian method under the entropy loss function outperforms others, achieving the lowest AEQL and a PCI of 1 for $m = 10$ and 30 , which confirms its effectiveness in detecting process shifts.

Table 6 presents the ARL and SDRL values obtained from both classical and Bayesian methods for inspection unit sizes of $10, 30$, and 50 , using the hyperparameters $(\eta, \kappa) = (1, 1)$. For $m = 10$, the Bayesian method under the squared error loss function consistently yields lower ARL_1 and $SDRL_1$ values for shift sizes ≤ 3.0 , indicating faster and more stable detection of moderate shifts. In contrast, the entropy-based Bayesian method shows superior performance

for larger shift sizes (> 3.0), with consistently lower ARL_1 and $SDRL_1$ values. When $m=30$, the Bayesian method under the squared error loss function outperforms all other approaches across the full range of shift magnitudes. Similarly, for $m=50$, the squared error-based Bayesian method remains superior across most shift sizes. However, the entropy-based Bayesian method demonstrates comparable performance, providing the same ARL_1 values for specific shifts (e.g., $\delta = 0.7, 1.0$, and 2.5 to 20.0). Overall, the performance of the two Bayesian approaches is nearly equivalent. These findings are further reinforced by

the AEQL and PCI metrics presented in Table 6, which provide additional evidence supporting the superiority of the squared error-based Bayesian method as the most effective approach overall. The consistently lowest AEQL values, combined with a PCI score of 1, demonstrate the method's optimal performance in detecting process shifts accurately. This indicates that the Bayesian chart not only efficiently identifies shifts but also minimizes false alarms, enhancing reliability in quality monitoring. Moreover, it shows adaptability across different shift sizes, confirming its practical utility in process control.

TABLE IV
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 4, \kappa = 1$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	374.0291	373.5288	374.3794	373.8790	372.6769	372.1766
	0.1	346.8654	346.3650	336.6631	336.1627	334.2948	333.7944
	0.3	298.9712	298.4708	274.2495	273.7491	272.4909	271.9904
	0.5	255.1478	254.6473	224.1403	223.6398	221.7441	221.2435
	0.7	215.5928	215.0922	182.6999	182.1992	180.3041	179.8034
	1.0	167.0558	166.5551	137.3923	136.8913	135.0505	134.5495
	1.5	108.2081	107.7070	86.4108	85.9093	84.7235	84.2220
	2.0	71.8726	71.3709	56.7700	56.2678	55.6490	55.1467
	2.5	48.3154	47.8128	38.2736	37.7702	37.4732	36.9698
	3.0	33.4962	32.9924	26.7856	26.2808	26.2124	25.7076
	5.0	9.9816	8.4815	8.3226	9.4684	7.9658	7.8066
	10.0	2.0069	1.4216	1.8820	1.2884	1.8632	1.2682
	20.0	1.0277	0.1686	1.0231	0.1536	1.0225	0.1516
AEQL		181.7348		155.2518		152.6536	
PCI		1.1905		1.0170		1.0000	
30	0.0	373.8466	373.3463	371.0624	370.5620	371.9160	371.4156
	0.1	336.5040	336.0036	332.3053	331.8049	332.7728	332.2725
	0.3	276.6035	276.1031	270.9445	270.4441	271.2822	270.7818
	0.5	225.3171	224.8165	219.3272	218.8266	219.5730	219.0724
	0.7	185.3974	184.8967	179.6306	179.1299	179.7501	179.2494
	1.0	139.0636	138.5627	134.1871	133.6862	134.2443	133.7434
	1.5	88.6156	88.1141	85.3213	84.8198	85.3337	84.8322
	2.0	58.4999	57.9977	56.4420	55.9397	56.4455	55.9432
	2.5	39.8367	39.3335	38.4874	37.9841	38.4882	37.9849
	3.0	28.0098	27.5052	27.1238	26.6191	27.1240	26.6193
	5.0	9.0074	8.4927	8.7888	8.2737	8.7888	8.2737
	10.0	1.9647	1.3767	1.9452	1.3560	1.9452	1.3560
	20.0	1.0256	0.1621	1.0249	0.1597	1.0249	0.1597
AEQL		160.0652		156.2573		156.2774	
PCI		1.0244		1.0000		1.0001	
50	0.0	372.0637	371.5633	370.6712	370.1709	369.1574	368.6571
	0.1	333.0274	332.5270	331.9314	331.4310	330.5064	330.0060
	0.3	270.4564	269.9559	269.2414	268.7410	268.0962	267.5957
	0.5	220.0230	219.5225	218.6417	218.1412	217.7407	217.2401
	0.7	180.4901	179.9894	179.2841	178.7834	178.5245	178.0238
	1.0	135.1861	134.6852	134.2863	133.7853	133.7225	133.2216
	1.5	86.0073	85.5059	85.4257	84.9242	85.0712	84.5698
	2.0	56.8091	56.3069	56.4155	55.9133	56.1892	55.6870
	2.5	38.9968	38.4935	38.7717	38.2684	38.6297	38.1264
	3.0	27.4763	26.9717	27.3346	26.8299	27.2341	26.7294
	5.0	8.9457	8.4309	8.9115	8.3966	8.8843	8.3694
	10.0	1.9661	1.3782	1.9636	1.3755	1.9605	1.3723
	20.0	1.0255	0.1616	1.0253	0.1612	1.0252	0.1609
AEQL		157.5778		156.9106		156.4642	
PCI		1.0071		1.0029		1.0000	

Table 7 shows the ARL and SDRL results under the hyperparameter setting $(\eta, \kappa) = (1, 2)$, again for inspection unit sizes 10, 30, and 50. For $m = 10$, the entropy-based Bayesian method consistently provides lower ARL_1 and $SDRL_1$ values across all shift magnitudes, indicating enhanced sensitivity in detecting deviations. At $m = 30$, both the squared error and entropy-based Bayesian methods outperform the classical approach across all shift sizes. For $m = 50$, both Bayesian estimators maintain superior

performance, with the entropy-based method showing marginally better ARL_1 results at specific shifts (e.g., $\delta = 0.7$), though the overall detection effectiveness remains comparable between the two.

As with the previous case, the AEQL and PCI results in Table 7 confirm the strength of the Bayesian approach under both loss functions. The consistently low AEQL values and PCI score of 1 demonstrate their efficiency in early shift detection and overall process monitoring reliability.

TABLE V
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 5, \kappa = 1$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	374.1073	373.6070	372.7186	372.2183	375.1323	374.6320
	0.1	345.9724	345.4721	334.6670	334.1666	336.0331	335.5327
	0.3	297.9898	297.4894	273.3058	272.8053	273.5030	273.0025
	0.5	254.5704	254.0699	224.5253	224.0247	223.7195	223.2189
	0.7	216.1083	215.6078	185.7903	185.2897	184.4911	183.9904
	1.0	166.2965	165.7957	137.9774	137.4765	136.5179	136.0170
	1.5	108.5875	108.0863	87.8020	87.3005	86.6632	86.1618
	2.0	71.6058	71.1040	57.1108	56.6086	56.4000	55.8978
	2.5	48.4019	47.8993	38.7987	38.2955	38.3581	37.8548
	3.0	33.4424	32.9386	27.0817	26.5770	26.7889	26.2842
	5.0	9.9730	9.4598	8.5351	8.0195	8.4861	7.9704
	10.0	2.0070	1.4216	1.8849	1.2915	1.8814	1.2877
	20.0	1.0276	0.1684	1.0231	0.1538	1.0230	0.1534
AEQL		181.6470		156.6484		155.5186	
PCI		1.1680		1.0073		1.0000	
30	0.0	368.2736	367.7733	370.0605	369.5601	370.7731	370.2727
	0.1	333.5519	333.0515	333.0034	332.5030	333.5734	333.0731
	0.3	272.5904	272.0899	269.6857	269.1853	270.1381	269.6376
	0.5	223.0860	222.5854	218.8389	218.3384	218.9776	218.4771
	0.7	183.3336	182.8329	179.2217	178.7210	179.2731	178.7724
	1.0	138.2983	137.7974	134.6138	134.1129	134.5963	134.0953
	1.5	88.1212	87.6198	85.5711	85.0696	85.5257	85.0242
	2.0	58.3051	57.8029	56.6550	56.1528	56.6164	56.1142
	2.5	39.4740	38.9708	38.4424	37.9392	38.4108	37.9075
	3.0	27.8119	27.3073	27.1211	26.6164	27.1002	26.5955
	5.0	8.9521	8.4373	8.7957	8.2806	8.7897	8.2746
	10.0	1.9591	1.3707	1.9459	1.3567	1.9451	1.3558
	20.0	1.0254	0.1613	1.0249	0.1597	1.0249	0.1596
AEQL		159.2423		156.3604		156.2996	
PCI		1.0188		1.0004		1.0000	
50	0.0	371.2382	370.7379	374.8880	374.3877	375.0043	374.5040
	0.1	333.3930	332.8926	336.6139	336.1135	336.7669	336.2666
	0.3	271.0003	270.4998	273.1125	272.6121	273.2794	272.7790
	0.5	220.3446	219.8440	221.9204	221.4198	221.9842	221.4836
	0.7	180.3263	179.8256	181.3838	180.8831	181.4690	180.9683
	1.0	135.2181	134.7172	135.9285	135.4276	135.9584	135.4575
	1.5	86.0918	85.5903	86.4915	85.9901	86.5073	86.0058
	2.0	56.9576	56.4554	57.1945	56.6923	57.1999	56.6977
	2.5	38.9284	38.4252	39.0961	38.5929	39.0982	38.5949
	3.0	27.4293	26.9246	27.5352	27.0306	27.5382	27.0336
	5.0	8.9596	8.4448	8.9902	8.4754	8.9905	8.4758
	10.0	1.9649	1.3770	1.9682	1.3805	1.9682	1.3805
	20.0	1.0255	0.1617	1.0256	0.1620	1.0256	0.1620
AEQL		157.5975		158.1230		158.1394	
PCI		1.0000		1.0033		1.0034	

TABLE VI
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 1, \kappa = 1$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	372.4671	371.9677	371.5622	371.0618	373.8065	373.3061
	0.1	346.6125	346.1122	334.5848	334.0845	336.4435	335.9431
	0.3	298.7512	298.2507	272.8283	272.3278	274.1684	273.6680
	0.5	254.8812	254.3807	218.5178	218.0173	219.3355	218.8349
	0.7	214.6443	214.1437	176.6964	176.1957	177.4044	176.9036
	1.0	165.8326	165.3318	132.4039	131.9030	132.7006	132.1997
	1.5	108.5172	108.0161	83.0523	82.5507	83.2037	82.7022
	2.0	71.7036	71.2018	54.3491	53.8467	54.3753	53.8729
	2.5	48.2075	47.7048	36.6433	36.1397	36.6494	36.1459
	3.0	33.3079	32.8041	25.6632	25.1582	25.6636	25.1586
	5.0	9.9544	9.4411	8.1880	7.6717	8.1878	7.6715
	10.0	2.0083	1.4230	1.8560	1.2604	1.8559	1.2604
	20.0	1.0277	0.1687	1.0222	0.1506	1.0222	0.1506
AEQL		181.3768		151.1491		151.2618	
PCI		1.2000		1.0000		1.0007	
30	0.0	369.6315	369.1312	369.1741	368.6738	369.6852	369.1849
	0.1	334.9314	334.4310	333.1610	332.6606	333.5425	333.0421
	0.3	273.2156	272.7152	269.4206	268.9202	269.7358	269.2354
	0.5	223.9281	223.4275	218.5253	218.0247	218.7911	218.2905
	0.7	183.0845	182.5839	177.8090	177.3083	177.9937	177.4930
	1.0	137.6719	137.1710	133.0737	132.5728	133.2002	132.6992
	1.5	88.1656	87.6642	84.9248	84.4233	85.0029	84.5014
	2.0	57.8922	57.3900	55.8157	55.3134	55.8648	55.3626
	2.5	39.6697	39.1665	38.2682	37.7649	38.3062	37.8029
	3.0	27.8289	27.3244	26.8926	26.3879	26.9169	26.4121
	5.0	8.9533	8.4385	8.7342	8.2190	8.7412	8.2260
	10.0	1.9598	1.3715	1.9395	1.3499	1.9405	1.3509
	20.0	1.0253	0.1612	1.0246	0.1588	1.0247	0.1589
AEQL		159.1992		155.3939		155.5035	
PCI		1.0245		1.0000		1.0007	
50	0.0	371.0064	370.5060	368.0349	367.5345	368.0516	367.5513
	0.1	334.8287	334.3284	331.9315	331.4311	331.9452	331.4448
	0.3	270.1869	269.6865	267.8260	267.3255	267.8349	267.3344
	0.5	220.2459	219.7453	218.0697	217.5691	218.0806	217.5800
	0.7	180.5805	180.0798	178.6479	178.1472	178.6479	178.1472
	1.0	135.1915	134.6906	133.7161	133.2152	133.7161	133.2152
	1.5	85.9438	85.4423	84.9982	84.4967	84.9985	84.4970
	2.0	56.8511	56.3488	56.2713	55.7691	56.2714	55.7691
	2.5	38.9497	38.4464	38.5777	38.0743	38.5777	38.0744
	3.0	27.4711	26.9664	27.2357	26.7310	27.2357	26.7310
	5.0	8.9439	8.4290	8.8842	8.3693	8.8842	8.3693
	10.0	1.9666	1.3787	1.9608	1.3726	1.9608	1.3726
	20.0	1.0255	0.1617	1.0253	0.1610	1.0253	0.1610
AEQL		157.5624		156.4663		156.4666	
PCI		1.0070		1.0000		1.0000	

Table 8 reports the ARL and SDRL values obtained from both classical and Bayesian methods for inspection unit sizes of 10, 30, and 50, with hyperparameters set to (2,2). For unit size $m = 10$, the Bayesian method under the entropy loss function consistently yields lower ARL_1 and $SDRL_1$ values across all shift magnitudes. At unit size $m = 30$, the Bayesian method under both the squared error and entropy loss functions outperforms the classical method for all shift sizes. For unit size $m = 50$, the Bayesian method under the entropy loss function consistently provides lower ARL_1 and $SDRL_1$ values across all shift magnitudes, while the squared error loss function demonstrates comparable performance to the entropy loss function for shifts of 1.00 and above.

Similar to the previous case, the AEQL and PCI results in Table 8 highlight the effectiveness of the Bayesian approach under both loss functions. The consistently low AEQL values, together with a PCI score of 1, indicate its efficiency in early shift detection and the reliability of overall process monitoring. These metrics demonstrate that the Bayesian approach can consistently maintain high sensitivity to process changes while minimizing false alarms, making it a highly dependable tool for quality control applications across a range of shift sizes and operational conditions. Furthermore, the results underscore the versatility of the method, showing that it performs effectively under different loss function criteria, which reinforces its practical applicability in real-world process monitoring scenarios.

Table 9 presents the ARL and SDRL values for inspection unit sizes of 10, 30, and 50, with hyperparameters set to (2,1). For unit size $m = 10$, the Bayesian method under the entropy loss function generally achieves lower ARL₁ and SDRL₁ values across most shift magnitudes. However, at a shift size of 0.1, the squared error loss function provides the minimum ARL₁ and SDRL₁ values. At unit size $m = 30$, the Bayesian method under the squared error loss function outperforms the classical method for all shift sizes. Similarly, for unit size $m = 50$, the Bayesian method under the squared error loss function consistently yields lower

ARL₁ and SDRL₁ values across all shift magnitudes. Likewise, the AEQL and PCI results presented in Table 9 further reinforce the robustness of the Bayesian approach across both loss functions. The consistently low AEQL values, together with a perfect PCI score, confirm its capability for timely and accurate shift detection, as well as reliable overall process monitoring. These findings demonstrate that the Bayesian method can maintain high sensitivity to process changes while minimizing false alarms, highlighting its practical effectiveness and versatility in various process control scenarios.

TABLE VII
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 1, \kappa = 2$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	373.4621	372.9617	373.9023	373.4019	373.4660	372.9657
	0.1	348.1549	347.6545	336.2133	335.7129	335.8776	335.3773
	0.3	298.9675	298.4671	270.8875	270.3870	270.7827	270.2822
	0.5	253.7179	253.2175	218.1228	217.6223	217.8864	217.3859
	0.7	215.2248	214.7243	177.9946	177.4939	177.7389	177.2382
	1.0	166.4171	165.9164	132.6808	132.1799	132.4899	131.9890
	1.5	109.2043	108.7032	83.7052	83.2037	83.5733	83.0718
	2.0	71.7699	71.2681	54.2565	53.7541	54.1640	53.6617
	2.5	48.5212	48.0186	36.7527	36.2493	36.6916	36.1882
	3.0	33.2948	32.7910	25.6201	25.1152	25.5746	25.0695
	5.0	9.9609	9.4476	8.1824	7.6662	8.1668	7.6504
	10.0	2.0079	1.4225	1.8571	1.2616	1.8550	1.2594
	20.0	1.0277	0.1687	1.0222	0.1506	1.0221	0.1504
AEQL		181.7141		151.3045		151.1149	
PCI		1.2025		1.0013		1.0000	
30	0.0	369.9580	369.4576	369.9775	369.4772	369.9775	369.4772
	0.1	333.9540	333.4536	331.9403	331.4399	331.9403	331.4399
	0.3	272.5293	272.0288	268.3735	267.8730	268.3735	267.8730
	0.5	222.6891	222.1886	217.4560	216.9554	217.4560	216.9554
	0.7	183.1085	182.6078	177.5319	177.0312	177.5319	177.0312
	1.0	138.4203	137.9194	133.7744	133.2735	133.7744	133.2735
	1.5	87.8745	87.3731	84.5788	84.0773	84.5788	84.0773
	2.0	57.8775	57.3753	55.7785	55.2763	55.7785	55.2763
	2.5	39.5054	39.0022	38.1313	37.6280	38.1313	37.6280
	3.0	27.7823	27.2778	26.9013	26.3965	26.9013	26.3965
	5.0	8.9673	8.4526	8.7529	8.2377	8.7529	8.2377
	10.0	1.9608	1.3726	1.9416	1.3521	1.9416	1.3521
	20.0	1.0254	0.1615	1.0247	0.1591	1.0247	0.1591
AEQL		159.0998		155.3318		155.3318	
PCI		1.0243		1.0000		1.0000	
50	0.0	371.9729	371.4725	370.7995	370.2992	370.7995	370.2992
	0.1	334.5308	334.0304	333.1917	332.6914	333.1917	332.6914
	0.3	270.8660	270.3655	269.7860	269.2856	269.7860	269.2856
	0.5	220.2635	219.7630	219.0225	218.5219	219.0225	218.5219
	0.7	180.6627	180.1620	179.4060	178.9053	179.4058	178.9051
	1.0	134.7694	134.2684	133.8746	133.3736	133.8746	133.3736
	1.5	86.2632	85.7617	85.6871	85.1856	85.6871	85.1856
	2.0	56.9415	56.4393	56.5860	56.0838	56.5860	56.0838
	2.5	38.9474	38.4441	38.7195	38.2162	38.7195	38.2162
	3.0	27.4875	26.9829	27.3403	26.8356	27.3403	26.8356
	5.0	8.9373	8.4225	8.9077	8.3928	8.9077	8.3928
	10.0	1.9662	1.3783	1.9634	1.3753	1.9634	1.3753
	20.0	1.0255	0.1617	1.0254	0.1613	1.0254	0.1613
AEQL		157.6152		156.9644		156.9644	
PCI		1.0041		1.0000		1.0000	

V. INDUSTRIAL APPLICATION OF POISSON-BASED QUALITY MONITORING

This section presents a comparative performance evaluation of the classical and Bayesian c-charts, constructed using squared error and entropy loss functions. The illustration is based on a real-world scenario involving 24 glass bottles randomly selected from a batch of 500 units, where the number of seeds contained in each bottle is assumed to follow a Poisson distribution with a mean (λ) of 4 [25]. This assumption implies that the seeds are randomly and independently distributed across the bottles. The observed count data reflect the seed distribution per case,

forming the basis for assessing the ability of each control chart to detect shifts in the process.

Figure 1 illustrates the monitoring performance of the classical c-chart when applied to the observed process data. The first nonconforming observation that exceeds the control limits occurs at the 62nd inspection point, indicating a delayed detection of the process shift. This result underscores the limited sensitivity of the classical method in identifying early or moderate deviations, thereby emphasizing the need for more adaptive approaches—such as Bayesian methods—especially in quality-critical industrial applications.

TABLE VIII
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 2, \kappa = 2$

m	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	373.2308	372.7305	370.0718	369.5715	370.0717	369.5714
	0.1	346.7343	346.2340	333.3247	332.8243	333.3247	332.8243
	0.3	297.5962	297.0958	269.8853	269.3849	269.8852	269.3847
	0.5	254.2724	253.7719	218.2475	217.7469	218.2475	217.7469
	0.7	215.2171	214.7165	178.6731	178.1724	178.6730	178.1723
	1.0	165.8289	165.3282	131.4337	130.9328	131.4336	130.9327
	1.5	109.0155	108.5143	83.0964	82.5949	83.0964	82.5949
	2.0	71.1855	70.6838	53.8301	53.3278	53.8301	53.3278
	2.5	48.1408	47.6382	36.4711	35.9676	36.4710	35.9676
	3.0	33.2389	32.7351	25.5400	25.0350	25.5399	25.0349
	5.0	9.9978	9.4846	8.2283	7.7121	8.2283	7.7121
	10.0	2.0069	1.4215	1.8574	1.2620	1.8574	1.2620
	20.0	1.0276	0.1684	1.0221	0.1503	1.0221	0.1503
AEQL		181.2951		150.8883		150.8881	
PCI		1.2015		1.0000		1.0000	
30	0.0	371.4311	370.9308	370.0050	370.9308	370.0050	370.9308
	0.1	335.0493	334.5489	332.1870	331.6870	332.1870	331.6870
	0.3	273.2290	272.7285	268.1956	267.6951	268.1956	267.6951
	0.5	223.7794	223.2788	218.0343	217.5338	218.0343	217.5338
	0.7	184.6265	184.1258	178.7381	178.2374	178.7381	178.2374
	1.0	138.4620	137.9611	133.6256	133.1247	133.6256	133.1247
	1.5	88.1243	87.6229	84.8201	84.3186	84.8201	84.3186
	2.0	57.9196	57.4174	55.8041	55.3018	55.8041	55.3018
	2.5	39.4379	38.9347	38.0965	37.5932	38.0965	37.5932
	3.0	27.8396	27.335	26.9596	26.4549	26.9596	26.4549
	5.0	8.9686	8.4538	8.7534	8.2382	8.7534	8.2382
	10.0	1.9598	1.3715	1.9405	1.3509	1.9405	1.3509
	20.0	1.0253	0.1612	1.0246	0.1588	1.0246	0.1588
AEQL		159.2409		155.4386		155.4386	
PCI		1.0245		1.0000		1.0000	
50	0.0	372.1739	371.6736	369.0930	368.5927	369.0930	368.5927
	0.1	333.3113	332.8109	330.5316	330.0312	330.5313	330.0309
	0.3	270.2207	269.7203	267.6883	267.1878	267.6883	267.1878
	0.5	220.7573	220.2567	218.4577	217.9571	218.4577	217.9571
	0.7	180.4869	179.9862	178.6070	178.1063	178.6068	178.1061
	1.0	135.0976	134.5966	133.6380	133.1371	133.6380	133.1371
	1.5	86.0993	85.5970	85.1534	84.6520	85.1534	84.6520
	2.0	56.9404	56.4382	56.3474	55.8452	56.3474	55.8452
	2.5	38.8845	38.3813	38.4986	37.9954	38.4986	37.9953
	3.0	27.4746	26.9700	27.2309	26.7262	27.2309	26.7262
	5.0	8.9426	8.4278	8.8823	8.3674	8.8823	8.3674
	10.0	1.9659	1.3780	1.9605	1.3723	1.9605	1.3723
	20.0	1.0254	0.1616	1.0252	0.1609	1.0252	0.1609
AEQL		157.5752		156.4641		156.4641	
PCI		1.0071		1.0000		1.0000	

TABLE IX
ARL AND SDRL RESULTS FOR BAYESIAN ESTIMATION USING $\eta = 2, \kappa = 1$

<i>m</i>	SHIFT	FREQUENTIST		BAYES (Square Error)		BAYES (Entropy)	
		ARL	SDRL	ARL	SDRL	ARL	SDRL
10	0.0	368.6068	368.1065	370.3860	369.8856	370.1939	369.6936
	0.1	343.7372	343.2368	332.9796	332.4792	333.4320	332.9316
	0.3	294.1893	293.6889	267.5261	267.0256	267.5135	267.0130
	0.5	251.8658	251.3653	218.6442	218.1436	218.0087	217.5081
	0.7	213.3991	212.8986	178.0206	177.5199	177.7885	177.2878
	1.0	164.6253	164.1245	133.0057	132.5047	132.356	131.8550
	1.5	107.3801	106.8789	83.2524	82.7509	82.94724	82.4457
	2.0	71.0497	70.5479	54.3240	53.8217	54.0884	53.5861
	2.5	47.7049	36.4391	36.5717	36.0682	36.4391	35.9356
	3.0	32.8872	32.3834	25.6000	25.0950	25.5203	25.0153
	5.0	9.8984	9.3851	8.2293	7.7131	8.2161	7.6999
	10.0	1.9988	1.4129	1.9988	1.2620	1.8566	1.2611
	20.0	1.0274	0.1679	1.0222	0.1507	1.0222	0.1506
AEQL		180.0243		152.3372		150.9030	
PCI		1.1930		1.0095		1.0000	
30	0.0	370.8784	370.3781	369.5542	369.0539	369.7886	369.2882
	0.1	333.6751	333.1747	330.4848	329.9844	330.6910	330.1907
	0.3	272.7502	272.2498	268.0223	267.5219	268.1290	267.6285
	0.5	224.1125	223.6119	218.7539	218.2533	218.8031	218.3025
	0.7	183.5611	183.0605	178.0421	177.5413	178.1002	177.5995
	1.0	138.7502	138.2493	133.9983	133.4974	134.0207	133.5197
	1.5	87.9885	87.4871	84.7295	84.2280	84.7315	84.2300
	2.0	58.0790	57.5768	58.0790	55.5051	56.0087	55.5064
	2.5	39.5148	39.0116	38.1707	37.6673	38.1709	37.6676
	3.0	27.7712	27.2666	26.8868	26.3821	26.8869	26.3821
	5.0	8.9646	8.7547	8.7547	8.2396	8.7547	8.2396
	10.0	1.9610	1.3728	1.9420	1.3525	1.9420	1.3525
	20.0	1.0253	0.1610	1.0245	0.1587	1.0245	0.1587
AEQL		159.2416		156.1329		155.5021	
PCI		1.0240		1.0041		1.0000	
50	0.0	371.8104	371.3100	369.1132	368.6128	369.1299	368.6296
	0.1	333.4776	332.9772	330.5510	330.0506	330.5785	330.0781
	0.3	270.0928	269.5923	267.5688	267.0683	267.6132	267.1127
	0.5	219.5703	219.0698	217.2184	216.7179	217.2403	216.7397
	0.7	180.1737	179.673	178.2604	177.7597	178.2604	177.7597
	1.0	135.1043	134.6034	133.6679	133.167	133.6693	133.1684
	1.5	86.0657	85.5642	85.1324	84.6309	85.1324	84.6309
	2.0	56.9089	56.4067	56.3142	55.8119	56.3143	55.8120
	2.5	38.9307	38.4274	38.5239	38.0206	38.5239	38.0206
	3.0	27.5229	27.0183	27.2661	26.7614	27.2661	26.7614
	5.0	8.9517	8.4369	8.8924	8.3775	8.8924	8.3775
	10.0	1.9678	1.3800	1.9623	1.3741	1.9623	1.3741
	20.0	1.0254	0.1616	1.0252	0.1609	1.0252	0.1609
AEQL		157.6126		156.4847		156.4856	
PCI		1.0072		1.0000		1.0000	

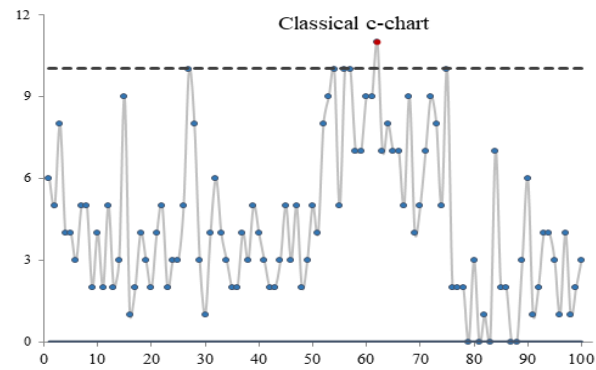


Fig 1. Detection of mean shifts using the classical method

In contrast, Figure 2 illustrates the monitoring performance of the Bayesian c-chart constructed under the squared error loss function. This chart successfully identifies nonconforming observations at the 27th, 54th, 56th, 57th, 62nd, and 75th inspection points, demonstrating its ability to detect process anomalies earlier than the classical approach. The earlier signaling of these deviations highlights the enhanced sensitivity and responsiveness of the squared error-based Bayesian method. Such performance improvement suggests a practical advantage in industrial quality control settings where early detection of subtle process changes is critical for minimizing defects, reducing waste, and maintaining consistent product standards.

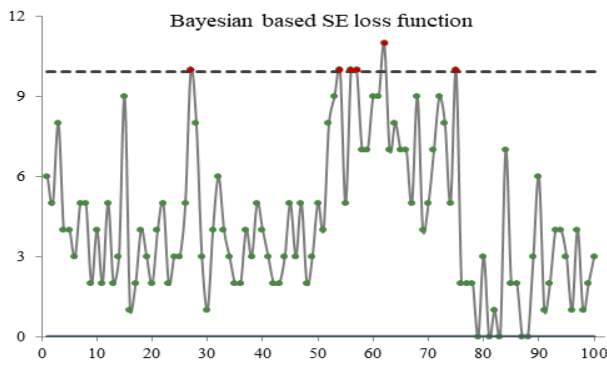


Fig 2. Detection of mean shifts using the squared error based Bayesian method

Figure 3 shows the results for the proposed Bayesian c-chart using the entropy loss function. Notably, the same set of nonconforming observations (27th, 54th, 56th, 57th, 62nd, and 75th) fall outside the control limits, as seen in the squared error loss case. This consistency reinforces the method's reliability and its enhanced ability to detect process changes more promptly than the classical approach. The agreement between the two loss functions further highlights the robustness of the Bayesian framework, offering stable and dependable monitoring performance in practice.

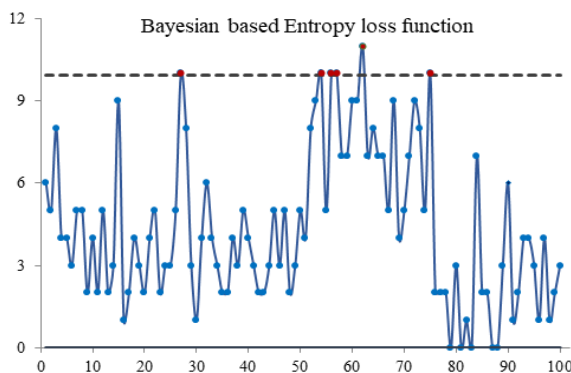


Fig 3. Detection of mean shifts using the entropy based Bayesian method

Overall, the comparative results clearly indicate that the Bayesian c-charts, under both loss functions, demonstrate superior sensitivity and efficiency in detecting process shifts. Their capability to identify anomalies at earlier stages of the monitoring sequence underscores their effectiveness as advanced tools for statistical quality control. This feature is particularly critical in industrial applications, where the prompt detection of defects is essential for ensuring process stability and safeguarding product quality. Furthermore, the consistent performance observed across different loss functions provides additional evidence of the robustness and practical applicability of the Bayesian approach.

VI. CONCLUSION

This study presents a comparative analysis of the performance of c-charts constructed using classical and Bayesian methods under two distinct loss functions: squared error and entropy. Performance was assessed using key indicators such as Average Run Length (ARL), Standard Deviation of Run Lengths (SDRL), Average Extra Quadratic Loss (AEQL), and the Performance Comparison Index (PCI).

The simulation results demonstrate that the Bayesian methods consistently outperform the classical approach, particularly for small and moderate inspection unit sizes. Among the Bayesian approaches, the entropy-based estimator frequently achieves the lowest ARL_1 and $SDRL_1$ values, reflecting superior sensitivity in detecting shifts. Both Bayesian methods yield PCI values close to 1, suggesting strong and reliable performance across different conditions.

To assess real-world applicability, the proposed methods were applied to actual process data involving the detection of foreign particles (seeds) in glass bottles. The results clearly indicate that the Bayesian c-charts—under both loss functions—exhibited superior sensitivity and efficiency in identifying process shifts compared to the classical method. Their ability to detect anomalies earlier in the monitoring sequence highlights their effectiveness as tools for quality control. This is particularly beneficial in industrial environments where early detection of defects is crucial to maintaining product quality and operational stability.

In conclusion, the Bayesian framework offers a flexible and powerful alternative for constructing c-charts, providing enhanced performance in both simulated and real-world scenarios. These findings support the broader adoption of Bayesian control charts in applications where accuracy and timeliness in detecting process changes are essential.

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