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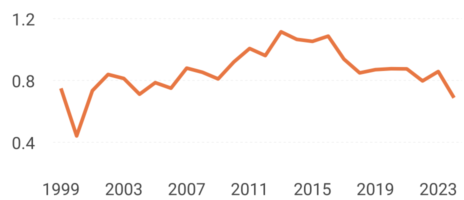

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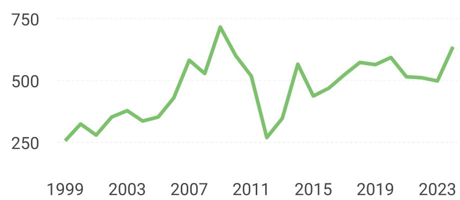

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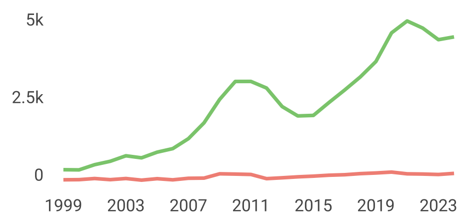

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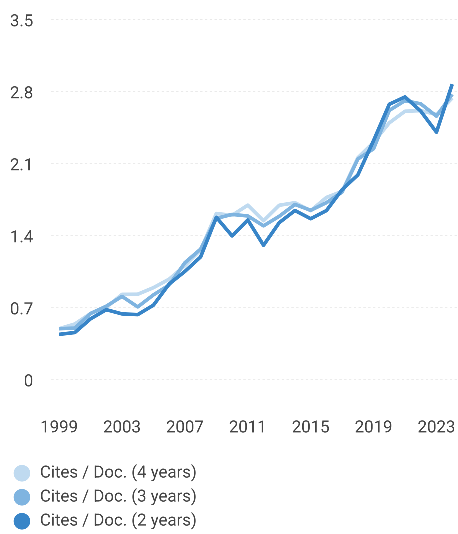

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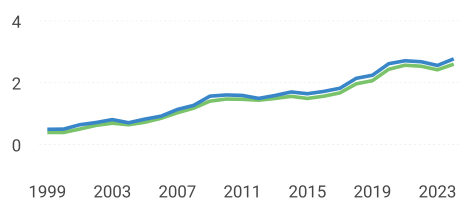

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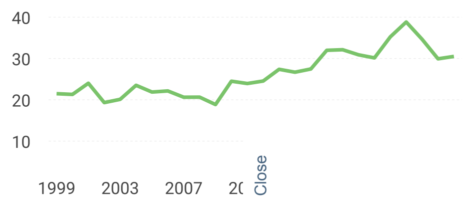

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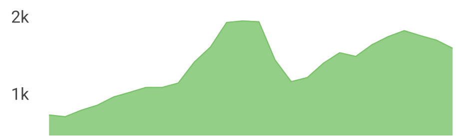

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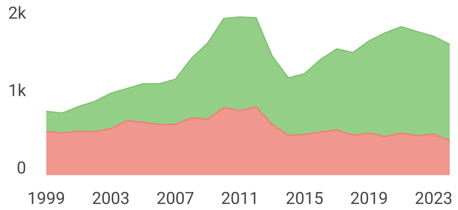

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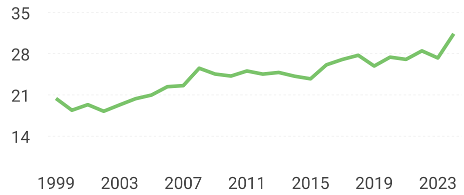

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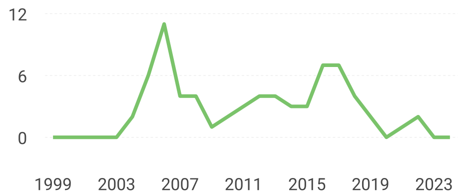

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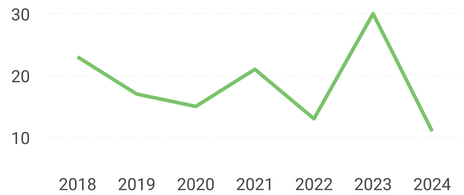

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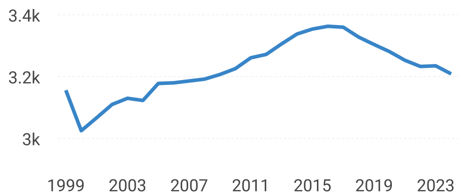

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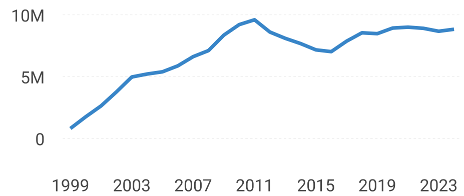

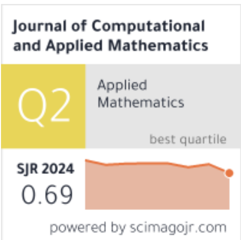

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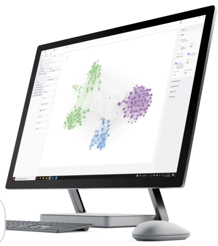

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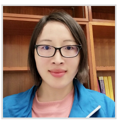

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New structured spectral gradient methods for nonlinear least squares with application in robotic motion control problems

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Robotic motion control

ABSTRACT

The recently introduced structured spectral Barzilai–Borwein-like (BB-like) gradient algorithms in (Optimization Methods and Software, 4(37), pp:1269–1288, 2022) which utilize substantial information of the Hessian matrix are efficient for solving nonlinear least squares (NLS) problems. However, a safeguarding technique is required for the spectral parameters in their formulation to be well-defined. In this paper, we present another spectral gradient algorithm that improves the efficiency of those formulations where the proposed structured spectral parameter does not necessarily require a safeguarding strategy. Moreover, with the aid of nonmonotone line search and some standard assumptions, we show the global convergence of the algorithm. In addition, the numerical results of the proposed algorithm on some benchmark problems are encouraging. Furthermore, we apply the algorithm to solving a motion control problem.

1. Introduction

Consider the nonlinear least square (NLS) problems, which is a special class of the general unconstrained optimization,

$$\min f(x), \qquad f(x) = \frac{1}{2}\sum_{i=1}^{m}[F_i(x)]^2 = \frac{1}{2}\|F(x)\|^2, \qquad x \in \mathbb{R}^n, \tag{1.1}$$

where for each $i = 1, 2, \ldots, m$, the residuals $F_i : \mathbb{R}^n \to \mathbb{R}$ is twice continuously differentiable functions which is bounded below. The problem (1.1) has recently received much attention due to its special structure. The gradient $g(x) = \nabla f(x)$ and the Hessian $H(x) = \nabla^2 f(x)$ of the objection function (1.1) are defined as follows:

$$g(x) = \sum_{i=1}^{m} F_i(x)\nabla F_i(x) = J(x)^T F(x), \tag{1.2}$$

$$H(x) = \sum_{i=1}^{m} \nabla F_i(x)\nabla F_i(x)^T + \sum_{i=1}^{m} F_i(x)\nabla^2 F_i(x) = J(x)^T J(x) + G(x), \tag{1.3}$$

respectively. $J(x)$ denotes the Jacobian matrix of the residual function F at x and $G(x) = \sum_{i=1}^{m} F_i(x)\nabla^2 F_i(x_k)$, where $F_i(x)$ is $i-$component of the residual vector $F(x)$ and $\nabla^2 F_i(x_k)$ is the Hessian matrix of $F_i(x)$, for each i.

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Computing the Hessian, $\nabla^2 F_i$, of the real valued-functions F_i, $i = 1, \ldots, m$, has been acknowledged to be a cumbersome task as well as costly process. Therefore, as an alternative, researchers usually find some efficient ways of approximating it with a keen interest in getting as much information about the objective function as possible. Moreover, the problem (1.1) is of particular interest to many researchers due to its appearance in several applications such as robotic motion control, data fitting, parameter estimation, imaging problems, stability and time delay-related problems, and so on [1–7].

The iterative scheme generally deployed to solve (1.1) is

$$x_{k+1} = x_k + h_k d_k, \quad k = 0, 1, 2, \ldots, \tag{1.4}$$

where x_k and x_{k+1} are the previous and current iterates, respectively, and d_k is a search direction usually required to satisfy the following descent condition $g_k^T d_k < 0$. The step length $h_k > 0$ is usually computed using suitable line search strategies. The line search strategy can be exact or inexact. The former is generally considered too expensive and therefore, researchers used the latter which requires relatively less computational effort. Please note that throughout this paper, every vector, say x, is a column vector while x^T denotes its transpose. One of the efficient strategies developed for computing the step length h_k is the inexact line search by Zhang and Hager [8] as stated in the following algorithm

Algorithm 1: The Zhang and Hager [8] line search.

Input : Objective $f(x)$, the search direction vector d_k at the point x_k and positive real numbers $\delta \in (0,1)$, $0 \leq \mu_{\min} \leq \mu_{\max} \leq 1$, $U_0 = f_0$, $\zeta_0 = \alpha_0^* = 1$, $W_0 = 1$ $\mu_k \in [\mu_{\min}, \mu_{\max}]$.

Step 1: Compute W_{k+1} and U_{k+1} using the following

$$U_0 = f(x_0) \quad and \quad U_{k+1} = \frac{\mu_k W_k U_k + f(x_{k+1})}{W_{k+1}}, \quad W_0 = 1 \quad and \quad W_{k+1} = \mu_k W_k + 1. \tag{1.5}$$

Step 2: Set $h = 1$, if

$$f(x_k + hd_k) \leq U_k + \delta h g_k^T d_k \tag{1.6}$$

then $h_k = h$. Else, set $h = h/2$ and test (1.6) again.

Popular methods for solving (1.1) include Newton's method, quasi-Newton methods, Gauss–Newton method, Levenberg–Marquardt method and Structured quasi-Newton methods (see [9–13]). Some of these methods utilize the special structure of the problem (1.1) while others do not [14,15]. The recent focus of researchers in this area deals with developing methods, for solving NLS problems, that are based on structured diagonal matrix approximation of the Hessian (1.3) and those that mimic conjugate gradient methods [16–18] as well as spectral gradient methods [19–21]. For example, Mohammad and Santos [22] coined a diagonal Hessian approximation method by approximating both the first term and the second term of the Hessian (1.3) in such a way that the structured secant condition, $H_k s_{k-1} \approx y_{k-1}$, ($s_{k-1}, y_{k-1}$ are given vectors) is fulfilled. However, to ensure sufficient decency of the search directions generated by their algorithm, they employed safeguarding methodologies that contain several user-defined parameters. This will certainly make their proposed search direction depend on user-defined parameters or at least be influenced by them. This is a sort of deficiency. To ameliorate some of the shortcomings in [22], Yahaya et al. [3] proposed structured quasi-Newton-based algorithms for solving (1.1) based on two formulations of the approximation of the Hessian (1.3). These two approximations were then used to construct two diagonal updating formulas for generating the search directions. Interestingly, unlike the method in [22], these algorithms require fewer user-defined parameters.

On the other hand, the structured spectral gradient-based approaches, for solving NLS problems, approximate the Hessian matrix (1.3) with a scalar multiple of an identity matrix where the scalar is usually updated in every iteration. Some of the proposed algorithms in this direction include the work of Mohammad and Waziri [19] which is based on the same structured vector used in [20]. Moreover, the authors employed a safeguarding strategy to ensure the search direction defined by those two structured parameters satisfied the descent condition. To improve upon the algorithms in [19], Awwal et al. [20] proposed another three structured spectral gradient algorithms where unlike in [19], they considered approximating only the second term of (1.3) with higher-order Taylor polynomial and retaining the exact structure of the first term. This means the algorithms in [20] utilize more information of (1.3). However, despite the advantages of these algorithms, their formulations still require safeguarding techniques to avoid negative curvature directions. To mitigate this shortcoming, we proposed a new structured spectral gradient algorithm that *does not* require any safeguard. As safeguarding is completely avoided in the definition of the structured spectral parameter, it will have more freedom to utilize the information gained from the preceding iteration. What follows is the summary of the contribution of this research article:

- A new structured spectral gradient algorithm is proposed.
- The proposed algorithm does not require any safeguarding technique.
- The global convergence of the proposal is shown under some standard assumptions.
- The algorithm is applied to solve the motion control problem of a robotic arm.

The rest of the article is subdivided into the following sections: In the next section, the formulation of the new structured spectral gradient algorithm and its convergence are outlined. More so, in Section 3, the numerical experiments are presented with some comparison and application. Finally, in Section 4, the conclusion of this research is presented.

2. Proposed NSSGM and its convergence

Consider the relation given by (1.3). Now, suppose that at a certain iteration say, k, we have

$$G(x_k) = \sum_{i=1}^{m} F_i(x_k)\nabla^2 F_i(x_k), \tag{2.1}$$

where $F_i(x_k)$ is $i-$component of the residual vector $F(x_k)$, and $\nabla^2 F_i(x_k)$ is the Hessian matrix of $F_i(x_k)$. Let $g_i(x_k)$ denotes the gradient of $F_i(x_k)$, for each i, we seek to construct some estimates for $G(x_k)$, say $D(x_k)$ such that the following secant equation,

$$D(x_k)s_{k-1} \approx G(x_k)s_{k-1} = \bar{y}_{k-1}, \tag{2.2}$$

is satisfied, where s_k is the difference between any two successive estimates of the solution and $\bar{y}$ is a structured vector to be determined. This means that substituting (2.2) in (1.3), gives the following secant equation

$$H(x_k)s_{k-1} = J(x_k)^T J(x_k)s_{k-1} + \bar{y}_{k-1}. \tag{2.3}$$

For simplicity, let G_k, J_k, and D_k denote $G(x_k)$, $J(x_k)$, and $D(x_k)$, respectively, with the $g_k^i = g_i(x_k)$ representing the gradient of the ith component of the residual $F(x_k)$.

Now, we approximate $G(x_{k+1})$ by employing a similar approach as presented in [17]. Since F_i is a real-valued function, then consider the higher order Taylor's series expansion of F_k^i as follows:

$$\begin{aligned} F_i(x_{k-1}) = {} & F_i(x_k) + \nabla F_i(x_k)^T(x_{k-1} - x_k) + \frac{1}{2}(x_{k-1} - x_k)^T \nabla^2 F_i(x_k)(x_{k-1} - x_k) \\ & + \frac{1}{6}(x_{k-1} - x_k)^T (T_k^i(x_{k-1} - x_k))(x_{k-1} - x_k) + O(\|x_{k-1} - x_k\|^4). \end{aligned} \tag{2.4}$$

Multiplying both sides of (2.4) by $(x_{k-1} - x_k)^T$ gives

$$\begin{aligned} (x_{k-1} - x_k)^T \nabla F_i(x_{k-1}) = {} & (x_{k-1} - x_k)^T \nabla F_i(x_k) - (x_{k-1} - x_k)^T \nabla^2 F_i(x_k)(x_{k-1} - x_k) \\ & + \frac{1}{2}(x_{k-1} - x_k)^T (T_k^i(x_{k-1} - x_k))(x_{k-1} - x_k) + O(\|x_{k-1} - x_k\|^4), \end{aligned} \tag{2.5}$$

where T_k^i is the tensor of $F_i(x_k)$, $i = 1, 2, \dots, m$. By setting $s_{k-1} = x_k - x_{k-1}$, then (2.4) and (2.5) respectively become

$$F_i(x_{k-1}) = F_i(x_k) - \nabla F_i(x_k)^T s_{k-1} + \frac{1}{2}s_{k-1}^T \nabla^2 F_i(x_k)s_{k-1} + \frac{1}{6}s_{k-1}^T (T_k^i s_{k-1})s_{k-1} + O(\|s_{k-1}\|^4), \tag{2.6}$$

and

$$s_{k-1}^T \nabla F_i(x_{k-1}) = s_{k-1}^T \nabla F_i(x_k) - s_{k-1}^T \nabla^2 F_i(x_k)s_{k-1} + \frac{1}{2}s_{k-1}^T (T_k^i s_{k-1})s_{k-1} + O(\|s_{k-1}\|^4). \tag{2.7}$$

Now, adding (2.6) and (2.7) and truncating the term containing the tensor onward, we have

$$\begin{aligned} s_{k-1}^T \nabla^2 F_i(x_k)s_{k-1} \approx & (\nabla F_i(x_k) - \nabla F_i(x_{k-1}))^T s_{k-1} + 6(F_i(x_{k-1}) - F_i(x_k)) \\ & + 3(\nabla F_i(x_k) + \nabla F_i(x_{k-1}))^T s_{k-1}. \end{aligned} \tag{2.8}$$

Now, we consider a simple approximation of $\nabla^2 F_i(x_k)$. That is, if we require that $\nabla^2 F_i(x_k) \approx \alpha_i I$, where I is an identity matrix and α_i is a scalar for each i, then (2.8) becomes

$$\begin{aligned} s_{k-1}^T \nabla^2 F_i(x_k)s_{k-1} \approx & \alpha_i s_{k-1}^T s_{k-1} \approx (\nabla F_i(x_k) - \nabla F_i(x_{k-1}))^T s_{k-1} + 6(F_i(x_{k-1}) - F_i(x_k)) \\ & + 3(\nabla F_i(x_k) + \nabla F_i(x_{k-1}))^T s_{k-1}. \end{aligned} \tag{2.9}$$

Since the scalar $s_{k-1}^T s_{k-1} = \|s_{k-1}\|^2 \neq 0$, otherwise, the solution of the problem in question has been achieved. Then dividing (2.9) by the scalar $\|s_{k-1}\|^2$ gives

$$\alpha_i \approx \frac{(\nabla F_i(x_k) - \nabla F_i(x_{k-1}))^T s_{k-1} + 6(F_i(x_{k-1}) - F_i(x_k)) + 3(\nabla F_i(x_k) + \nabla F_i(x_{k-1}))^T s_{k-1}}{\|s_{k-1}\|^2}. \tag{2.10}$$

Therefore, the approximation of $\nabla^2 F_i(x_k)s_{k-1}$ is

$$\nabla^2 F_i(x_k)s_{k-1} \approx \frac{(\nabla F_i(x_k) - \nabla F_i(x_{k-1}))^T s_{k-1} + 6(F_i(x_{k-1}) - F_i(x_k)) + 3(\nabla F_i(x_k) + \nabla F_i(x_{k-1}))^T s_{k-1}}{\|s_{k-1}\|^2} s_{k-1}. \tag{2.11}$$

Now, substituting (2.11) into (2.1) gives

$$\begin{aligned} G_k s_{k-1} = & \frac{1}{\|s_{k-1}\|^2} \sum_{i=1}^{m} F_i(x_k)[(\nabla F_i(x_k) - \nabla F_i(x_{k-1}))^T s_{k-1} + 6(F_i(x_{k-1}) - F_i(x_k)) \\ & + 3(\nabla F_i(x_k) + \nabla F_i(x_{k-1}))^T s_{k-1}]s_{k-1}. \end{aligned} \tag{2.12}$$

Since $\nabla F_i(x) = J(x)$, then using (2.2), we have the following structured secant equation

$$D_k s_{k-1} = \bar{y}_{k-1}, \tag{2.13}$$

where

$$\bar{y}_{k-1} = (J_k - J_{k-1})^T F(x_k) + \frac{\vartheta_{k-1}}{\|s_{k-1}\|^2} s_{k-1}, \tag{2.14}$$

$$\vartheta_{k-1} = 3F(x_k)^T [(J_k - J_{k-1})s_{k-1} - 2(F(x_k) - F(x_{k-1}))]. \tag{2.15}$$

Finally, combining (2.3) and (2.13) yields

$$H_k s_{k-1} = \gamma_{k-1}, \tag{2.16}$$

where

$$\gamma_{k-1} = J_k^T J_k s_{k-1} + (J_k - J_{k-1})^T F(x_k) + \frac{\vartheta_{k-1}}{\|s_{k-1}\|^2} s_{k-1}. \tag{2.17}$$

Recently, as mentioned in the previous section, Awwal et al. [20] proposed three structured spectral gradient algorithms by incorporating the structured vector (2.17) into the BB spectral parameters as well as their convex combination. They defined the search directions as follows

$$d_k^{(i)} = -\lambda_k^{(i)} g_k, \quad i = 1, 2, 3, \quad \text{and} \quad k \geq 1, \tag{2.18}$$

where

$$\lambda_k^{(i)} = \begin{cases} \frac{\|s_{k-1}\|^2}{s_{k-1}^T \gamma_{k-1}}, & \text{for} \quad i = 1, \\ \frac{s_{k-1}^T \gamma_{k-1}}{\|\gamma_{k-1}\|^2}, & \text{for} \quad i = 2, \\ r_k \frac{\|s_{k-1}\|^2}{\gamma_{k-1}^T s_{k-1}} + (1 - r_k) \frac{\gamma_{k-1}^T s_{k-1}}{\|\gamma_{k-1}\|^2}, & \text{for} \quad i = 3, \end{cases} \tag{2.19}$$

with the scalar $r_k \in [0, \ 1]$. To avoid negative curvature directions, the authors replaced $s_{k-1}^T \gamma_{k-1}$ with the following safeguard strategy

$$\tau_k^{(i)} = \max\{\beta \lambda_{k-1}^{(i)}, \ 2|s_{k-1}^T \gamma_{k-1}| + \|s_{k-1}\|^2 + \|\gamma_{k-1}\|^2\}, \quad \beta > 0, \tag{2.20}$$

whenever it is nonpositive, where $i = 1, 2, 3$.

Now, using Cauchy Schwarz inequality on $\lambda_k^{(1)}$ and $\lambda_k^{(2)}$, we have

$$\lambda_k^{(1)} = \frac{\|s_{k-1}\|^2}{s_{k-1}^T \gamma_{k-1}} \geq \frac{\|s_{k-1}\|}{\|\gamma_{k-1}\|}, \tag{2.21}$$

and

$$\lambda_k^{(2)} = \frac{s_{k-1}^T \gamma_{k-1}}{\|\gamma_{k-1}\|^2} \leq \frac{\|s_{k-1}\|}{\|\gamma_{k-1}\|}. \tag{2.22}$$

This means that $\lambda_k^{(1)} \geq \lambda_k^{(2)}$ for all k. That is, the quantity $\lambda_k^{(1)} - \lambda_k^{(2)}$ is nonnegative.

Motivated by this, we define the search direction of the new algorithm as

$$d_k = -\psi_k g_k, \quad k \geq 1, \tag{2.23}$$

where

$$\psi_k = \frac{\|s_{k-1}\|}{\|\gamma_{k-1}\|} + \frac{\|s_{k-1}\|^2}{s_{k-1}^T \gamma_{k-1}} - \frac{s_{k-1}^T \gamma_{k-1}}{\|\gamma_{k-1}\|^2}. \tag{2.24}$$

To analyze the advantage of the parameter (2.24) as well as the convergence of the proposed algorithm, we require the following standard assumption.

Assumption 2.1. The following standard assumptions are useful in the convergence analysis of the proposed method.

A1. The level set $\mathcal{D} = \{x \in \mathbb{R}^n \mid f(x) \leq f(x_0)\}$ is bounded. That is, $\|x\| \leq \omega$, holds for all $x \in \mathcal{D}$, where $\omega > 0$.

A2. There exist constants $L_1 > 0$ and $L_2 > 0$ such that for all $x, y \in \mathcal{D}$, we have

$$\|J(x) - J(y)\| \leq L_1 \|x - y\|, \tag{2.25}$$

$$\|F(x) - F(y)\| \leq L_2 \|x - y\|. \tag{2.26}$$

The two inequalities (2.25) and (2.26) give rise to the following conclusions

$$\|g(x) - g(y)\| \leq l\|x - y\|, \quad \|F(x)\| \leq \omega_1, \quad \|J(x)\| \leq \omega_2, \quad \|g(x)\| \leq \gamma_3,$$

where l, ω_1, ω_2 and ω_3 are positive constants.

Lemma 2.2. *Suppose that Assumption 2.1 (A2) holds. Let the structured vector γ_{k-1} be defined by (2.17), then there exists some positive constant, say $M > 0$, such that*

$$\|\gamma_{k-1}\| \le M\|s_{k-1}\|, \quad \forall\ k. \tag{2.27}$$

$$\psi_k \ge 1/M, \quad \forall\ k. \tag{2.28}$$

Proof. Let $\omega \in [0,\ 1]$, by Assumption 2.1 and mean value theorem, we have

$$\begin{aligned}
|\vartheta_{k-1}| &= \left|3F(x_k)^T\left[(J_k - J_{k-1})s_{k-1} - 2(F(x_k) - F(x_{k-1}))\right]\right| \\
&= \left|3F(x_k)^T\left[(J_k - J_{k-1})s_{k-1} - 2J(x_{k-1} + \omega s_{k-1})s_{k-1}\right]\right| \\
&\le 3\|F(x_k)\|\,\|(J_k - J_{k-1}) - 2J(x_{k-1} + \omega s_{k-1})\|\,\|s_{k-1}\| \\
&\le 3\|F(x_k)\|\|s_{k-1}\|\left[\|J_k - J(x_{k-1} + \omega s_{k-1})\| + \|J_{k-1} - J(x_{k-1} + \omega s_{k-1})\|\right] \\
&\le 3\|F(x_k)\|\|s_{k-1}\|\left[L_1(1-\omega)\|s_{k-1}\| + L_1\omega\|s_{k-1}\|\right] \\
&\le 6L_1\omega_1\|s_{k-1}\|^2.
\end{aligned} \tag{2.29}$$

If we let $m := 6L_1\omega_1$, we obtain

$$|\vartheta_{k-1}| \le m\|s_{k-1}\|^2, \quad \forall\ k. \tag{2.30}$$

Again, using Assumption 2.1 (A2) and (2.30), we obtain

$$\begin{aligned}
\|\gamma_{k-1}\| &= \left\|J_k^TJ_ks_{k-1} + \left[(J_k - J_{k-1})^TF(x_k) + \frac{\vartheta_{k-1}}{\|s_{k-1}\|^2}s_{k-1}\right]\right\| \\
&\le \|J_k\|^2\|s_{k-1}\| + \|J_k - J_{k-1}\|\|F(x_k)\| + \frac{|\vartheta_{k-1}|}{\|s_{k-1}\|^2}\|s_{k-1}\| \\
&\le \|J_k\|^2\|s_{k-1}\| + \|J_k - J_{k-1}\|\|F(x_k)\| + \frac{m\|s_{k-1}\|^2}{\|s_{k-1}\|^2}\|s_{k-1}\| \\
&\le \omega_2^2\|s_{k-1}\| + \omega_1L_1\|s_{k-1}\| + m\|s_{k-1}\| \\
&= (\omega_2^2 + \omega_1L_1 + m)\|s_{k-1}\|.
\end{aligned} \tag{2.31}$$

By letting $M := \omega_2^2 + \omega_1L_1 + m$, we get the desired result. Lastly, from (2.21) and (2.22), we have that $\frac{\|s_{k-1}\|^2}{s_{k-1}^T\gamma_{k-1}} - \frac{s_{k-1}^T\gamma_{k-1}}{\|\gamma_{k-1}\|^2} \ge 0$, and therefore using (2.27) $\psi_k \ge \frac{\|s_{k-1}\|}{\|\gamma_{k-1}\|} \ge \frac{\|s_{k-1}\|}{M\|s_{k-1}\|} = \frac{1}{M}$. ■

From Lemma 2.2, we see that the spectral parameter ψ_k is strictly positive, for all k.

Algorithm 2: New Structured Spectral Gradient Method (NSSGM)

Input : Initial approximation $x_0 \in \mathrm{dom}(f)$, $0 << \psi_{\max} << +\infty$ and $Tol > 0$.
Step 0: Compute $f(x_0)$ and $d_0 = -g_0$. Set $k = 0$.
Step 1: Compute $F(x_k)$ and g_k. If $\|g_k\| \le Tol$ or $k \ge k_{\max}$, stop.
Step 2: Compute h_k using Algorithm 1.
Step 3: Update the next iterate using $x_{k+1} = x_k + h_kd_k$.
Step 4: Compute $\gamma_{k-1} = J_k^TJ_ks_{k-1} + (J_k - J_{k-1})^TF(x_k) + \frac{\vartheta_{k-1}}{\|s_{k-1}\|^2}s_{k-1}$,
where $\vartheta_{k-1} = 3F(x_k)^T[(J_k - J_{k-1})s_{k-1} - 2(F(x_k) - F(x_{k-1}))]$.
Step 5: Update the search direction

$$d_k = -\hat{\psi}_kg_k, \tag{2.32}$$

$$\hat{\psi}_k = \min\left\{\frac{\|s_{k-1}\|}{\|\gamma_{k-1}\|} + \frac{\|s_{k-1}\|^2}{s_{k-1}^T\gamma_{k-1}} - \frac{s_{k-1}^T\gamma_{k-1}}{\|\gamma_{k-1}\|^2}, \psi_{\max}\right\} \tag{2.33}$$

Step 6: Set $k := k + 1$ and go to step 1.

Remark 2.3. From the definition of the spectral parameter (2.33) and Lemma 2.2, we see that $\hat{\psi}_k$ is automatically bounded, i.e.

$$\frac{1}{M} \le \hat{\psi}_k \le \psi_{\max}, \quad \forall\ k. \tag{2.34}$$

This means that unlike the methods in [20], we do not require any special safeguarding strategy for the spectral parameter $\hat{\psi}_k$ defined by (2.33). In addition, it is not difficult to see that the following hold with regards to the search direction defined by (2.32)

$$g_k^Td_k \le -\frac{1}{M}\|g_k\|^2. \tag{2.35}$$

$$\|d_k\| \leq \psi_{\max}\|g_k\|. \tag{2.36}$$

Lemma 2.4. *Let $\delta \in (0,1)$ and suppose d_k is the search direction defined by (2.32) such that the inequalities (2.35) and (2.36) hold. Suppose Assumption 2.1 holds, then the Algorithm 2 (NSSGM) is well-defined.*

Proof. Let g_k be the gradient of the objective function (1.1) generated by Algorithm 2 (NSSGM) at x_k such that $\|g_k\| \neq 0$. By the fact that (2.35) holds for the search direction (2.32) then by Assumption 2.1, there exists a step length h_k^* sufficiently small enough such that

$$f(x_k + h^* d_k) \leq U_k + \delta h^* g_k^T d_k, \tag{2.37}$$

holds, where the next iterate $x_{k+1} = x_k + h^* d_k$ is well-defined.

Suppose for contradiction that, at a certain iteration, there exists some $h_j \geq 0$ for which the line search (1.6) does not hold, then we have

$$f(x_k + h_j d_k) > U_k + \delta h_j g_k^T d_k, \quad \text{for all} \quad \alpha_j \geq 0, \tag{2.38}$$

where $\{h_j\}$ is a strictly decreasing sequence satisfying $\lim_{j\to\infty} h_j = 0$.

By the definition of the problem (1.1), we have $f(x_k) \geq 0$, $\forall k$. Since $U_0 = f(x_0)$, (see, Algorithm 1) and the fact that U_k is a convex combination of U_{k-1} and $f(x_k)$, it holds that $U_k \geq 0 \ \ \forall \ \ k$.

Now, since it holds from (2.36) that $\|d_k\| \leq \psi_{\max}\|g_k\|$, then (2.38) becomes

$$\begin{aligned}
U_k &< f(x_k + h_j d_k) - \delta h_j g_k^T d_k \\
&\leq f(x_k + h_j d_k) + \delta h_j \|g_k\|\|d_k\| \\
&\leq f(x_k + h_j d_k) + \delta \psi_{\max} h_j \|g_k\|^2 \\
&\leq f(x_k + h_j d_k) + \delta \psi_{\max} h_j \gamma_3^2.
\end{aligned}$$

This means that taking the limit on both sides as $j \to \infty$ gives

$$U_k \leq f(x_k). \tag{2.39}$$

However, since

$$U_k = \frac{\mu_{k-1} W_{k-1} U_{k-1} + f(x_k)}{\mu_{k-1} W_{k-1} + 1},$$

it means that U_k lies between U_{k-1} and $f(x_k)$. Merging this with (2.39) yields $f(x_k) = U_k$. This further means that $\mu_{k-1} = 0$, since $U_{k-1} \neq 0$, and $W_{k-1} \neq 0$. Therefore, the non-monotone line search turns into monotone. Thus, (2.38) turns to

$$f(x_k + h_j d_k) > f(x_k) + \delta h_j g_k^T d_k, \tag{2.40}$$

this implies,

$$\frac{f(x_k + h_j d_k) - f(x_k)}{h_j} > \delta g_k^T d_k.$$

Now, taking limit as $j \to \infty$ and using Assumption 2.1 gives $g_k^T d_k \geq \delta g_k^T d_k$. Since $g_k^T d_k < 0$, it must hold that $\delta \geq 1$, which is a contradiction. Hence, the proof. ∎

The following result is from [8] and is useful in proving the convergence result of the proposed method.

Lemma 2.5. *Suppose the nonmonotone line search algorithm is employed with the search direction being descent such that $\|\nabla f(x) - \nabla f(x_k)\| \leq L\|x - x_k\|$ for all x on the line segment connecting x_k and $x_k + h_k \rho d_k$, $\rho > 0$, if $\rho h_k \leq \overline{\mu}$, $\overline{\mu} > 0$, then*

$$h_k \geq \min\left\{\frac{\overline{\mu}}{\rho}, \frac{2(1-\delta)}{L\rho}\frac{|g_k^T d_k|}{\|d_k\|^2}\right\}. \tag{2.41}$$

Now, we state the convergence result of the proposed method. The proof follows directly from Theorem 2.2 [8]. We only repeat it here for the benefit of the potential readers.

Theorem 2.6. *Let $\{x_k\}$ be the sequence generated by Algorithm 2 and suppose $f(x)$ is given by (1.1) such that Assumption 2.1 holds. Then the sequence of iterates $\{x_k\}$ is contained in the level set and the following conclusions hold:*

$$\lim_{k\to\infty} \inf \|g_k\| = 0, \quad \textit{and}$$

$$\lim_{k\to\infty} \|g_k\| = 0, \quad \textit{if} \ \ 0 < \mu_{\max} < 1.$$

Proof. It was shown in Lemma 1.1 of [8] that for W_{k+1} defined in Step 1 of Algorithm 1, we have

$$W_{k+1} = 1 + \sum_{i=0}^{k} \prod_{m=0}^{i} \mu_{k-m} \leq k+2. \tag{2.42}$$

Next, consider the following cases:

Case 1. If $\rho h_k \geq \overline{\mu}$, then $h_k \geq \frac{\overline{\mu}}{\rho}$. By (1.6) and (2.35), it follows that

$$\begin{aligned} f(x_{k+1}) &\leq U_k + \delta h_k g_k^T d_k \\ &\leq U_k - \frac{\delta}{M} h_k \|g_k\|^2 \\ &\leq U_k - \frac{\delta \overline{\mu}}{M\rho} \|g_k\|^2. \end{aligned}$$

Case 2. On the other hand, if $\rho h_k \leq \overline{\mu}$, then by (2.41),

$$h_k \geq \frac{2(1-\delta)}{L\rho} \frac{|g_k^T d_k|}{\|d_k\|^2}. \tag{2.43}$$

From (2.35) and (2.36), we deduce $|g_k^T d_k| \geq \frac{1}{M}\|g_k\|^2$ and $\frac{1}{\|d_k\|^2} \geq \frac{1}{\psi_{max}^2 \|g_k\|^2}$. Combining with (1.6) gives

$$\begin{aligned} f(x_{k+1}) &\leq U_k + \delta h_k g_k^T d_k \\ &\leq U_k - \frac{\delta}{M} \frac{2(1-\delta)}{L\rho} \frac{|g_k^T d_k|}{\|d_k\|^2} \|g_k\|^2 \\ &\leq U_k - \frac{2(1-\delta)}{L\rho} \frac{\delta}{M^2} \frac{\|g_k\|^2}{\|d_k\|^2} \|g_k\|^2 \\ &\leq U_k - \frac{2(1-\delta)}{L\rho \psi_{max}^2} \frac{\delta}{M^2} \|g_k\|^2, \end{aligned}$$

where the third and last inequalities follow from (2.35) and (2.36), respectively.

Setting $\upsilon = \min\left\{ \frac{\delta\overline{\mu}}{M\rho}, \frac{2(1-\delta)}{L\rho\psi_{max}^2} \frac{\delta}{M^2} \right\}$, yields

$$f(x_{k+1}) \leq U_k - \upsilon \|g_k\|^2. \tag{2.44}$$

Combining the relation (1.5) in Step 1 of Algorithm 1 and (2.44), gives

$$\begin{aligned} U_{k+1} &= \frac{\mu_k W_k U_k + f(x_{k+1})}{W_{k+1}} \\ &\leq \frac{\mu_k W_k U_k + U_k - \upsilon\|g_k\|^2}{W_{k+1}} \\ &= \frac{(\mu_k W_k + 1)U_k - \upsilon\|g_k\|^2}{W_{k+1}} \\ &= U_k - \frac{\upsilon\|g_k\|^2}{W_{k+1}}. \end{aligned} \tag{2.45}$$

From (2.44), we can deduce $f(x_{k+1}) \leq U_k$, and by the fact that the objective function f is bounded from below, we have that U_k is bounded from below, $\forall k$.

Now, from (2.45), we have that

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^2}{W_{k+1}} < \infty. \tag{2.46}$$

If $\|g_k\|$ were bounded away from 0, then (2.46) would be violated since $W_{k+1} \leq k+2$. Hence, $\lim_{k\to\infty} \inf \|g_k\| = 0$, holds.

If $\mu_{\max} < 1$, then by (2.42),

$$W_{k+1} = 1 + \sum_{j=0}^{k} \prod_{i=0}^{j} \mu_{k-i} \leq 1 + \sum_{j=0}^{k} \mu_{\max}^{j+1} \leq \sum_{j=0}^{\infty} \mu_{\max}^{j} = \frac{1}{1-\mu_{\max}}. \tag{2.47}$$

Combining with (2.46), we have $\lim_{k\to\infty} \|g_k\| = 0$, holds. ■

3. Numerical experiments and comparison

In this section, the numerical performance as well as computational efficiency of the proposed method shall be demonstrated. The experiment is divided into two subsections. The first subsection discusses the numerical performance of Algorithm 2 (NSSGM) on some benchmark test problems in comparison with two existing algorithms of similar characteristics. On the other hand, in the other subsection, Algorithm 2 is implemented to solve problems arising from 2D robotic motion control. All experiments are conducted on a personal computer with an Intel Core(TM) i5-8250u processor with 4 GB of RAM and a CPU 1.60 GHz.

Table 1
List of zero and nonzero test problems used for the experiment in Section 3.1.

S/N	Problem name	Initial points	Size	Reference
Zero and Nonzero test problems				
1	Penalty function I	$(3, 3, \dots, 3)^T$	Large	[23]
2	Variably dimension	$(1-1/n, 1-2/n, \dots, 0)^T$	Large	[24]
3	Trigonometric function	$(1, 1, \dots, 1)^T$	Large	[24]
4	Linear function–full rank	$(1, 1, \dots, 1)^T$	Large	[24]
5	Problem 202	$(2, 2, \dots, 2)^T$	Large	[25]
6	Problem 212	$(1/2, 1/2, \dots, 1/2)^T$	Large	[25]
7	Strictly convex function I	$(1/n, 1/n, \dots, 1/n)^T$	Large	[26]
8	Sin function II	$(1, 1, \dots, 1)^T$	Large	[27]
9	Exponential function I	$(n/n-1, n/n-1, \dots, n/n-1)^T$	Large	[23]
10	Exponential function II	$(1/n^2, 1/n^2, \dots, 1/n^2)^T$	Large	[23]
11	Logarithmic function I	$(1, 1, \dots, 1)^T$	Large	[23]
12	Trigonometric exponential function	$(1/2, 1/2, \dots, 1/2)^T$	Large	[25]
13	Extended Powell function	$(1.5E-4, \dots, 1.5E-4)^T$	Large	[23]
14	Function 21	$(-1, -1, \dots, -1)^T$	Large	[23]
15	Extended Rosenbrock function	$(-1, -1, \dots, -1)^T$	Large	[24]
16	Extended Himmelblau function	$(-1, 1, -1, 1, \dots, -1, 1)^T$	Large	[28]
17	Function 27	$(100, \frac{1}{n^2}, \frac{1}{n^2}, \dots, \frac{1}{n^2})$	Large	[23]
18	Trigonometric logarithmic function	$(1, 1, \dots, 1)^T$	Large	[23]
19	Zero Jacobian function	for $i = 1$, $\frac{100(n-100)}{n}$, for $i \geq 2$, $\frac{(n-1000)(n-500)}{(60n)^2}$	Large	[23]
20	Exponential function	$(1/2, 1/2, \dots, 1/2)^T$	Large	[23]
21	Function 18	$(1, 1, \dots, 1)^T$	Large	[23]
22	Brown almost linear function	$(1/n, 1/n, \dots, 1/n)^T$	Large	[24]
23	Brown Badly Scaled function	$(1, 1)^T$	Small	[24]
24	Jennrich and Sampson function	$(1, 1)^T$	Small	[24]
25	Box three-dimensional function	$(1, 1, 1)^T$	Small	[24]
26	Rank deficient function	$(1, 1)^T$	Small	[24]
27	Rosenbrock function	$(1, 1)^T$	Small	[24]
28	Parameterized problem	$(1, 1)^T$	Small	[29]
29	Freudenstein and Roth function	$(1, 1)^T$	Small	[24]
30	Beale Function	$(1, 1)^T$	Small	[24]

3.1. Numerical performance on benchmark test problems

The first efficiency test for NSSGM is done by implementing it to solve some benchmark test problems and then comparing its numerical performance with the SSHBB algorithm developed in [20] and the SSGM2 proposed in [19]. The comparison test is done based on #iter (the number of iterations), #fval (the number of function evaluations), #nmvp (the number of matrix–vector products) and #time (the CPU time) recorded. *It is crucial to mention here that all the NSSGM, SSHBB, and SSGM2 are coded in MATLAB (R2019b) such that, for each test problem considered, the components of the structured spectral parameters are computed directly as a matrix–vector product without the need to explicitly form or store any matrix throughout the iteration process.* This means that the NSSGM, SSHBB, and SSGM2 are implemented as matrix-free algorithms.

The three algorithms are implemented using the same parameters as presented in [20]. In the course of this experiment, thirty (30) benchmark test problems, where twenty-two (22) are large scale and the remaining are small scale, were solved. The dimensions of the large-scale problems are varied as 3000, 9000, and 15 000. Details of the test problems are given in Table 1. During the iteration process, a method is declared to have achieved an approximate solution of a particular problem whenever $\|g_k\| \leq 10^{-6}$. However, if the number of iterations is in excess of 1000 iterations and the stopping criterion mentioned above has not been satisfied, then a failure is declared and is denoted as "_". The details of the numerical values recorded by each algorithm have been presented in Tables 2–4. Perusing Tables 2–4, it is very easy to note that the proposed NSSGM solves all the test problems considered, successfully, whereas, its competitors, SSHBB and SSGM2, failed in a number of cases. This suggests that the new NSSGM can be an alternative to the existing SSHBB and SSGM2 methods. Furthermore, although the numerical results in Tables 2–4 show that the three algorithms are competitive, we can confirm the relatively superior performance of NSSGM over SSHBB and SSGM2 as it solves all the test problems including those that could not be solved by others. This underscores the efficiency of the new NSSGM algorithm.

3.2. Application in 2D robotic motion control

Recently, applications of optimization algorithms to solve different types of problems are gaining more attention. One such application that is of interest to us, in this paper, is the 2-dimensional robotic motion control problem. In what follows, the new NSSGM is employed to track a two-joint planar robot manipulator. For a detailed description of the discrete-time kinematics equation of a two-joint planar robot manipulator, the reader may refer to the Ref. [30–33] and the references therein. The task at hand is to solve the following nonlinear least square problem:

$$\min_{F_k \in \mathbb{R}^2} \frac{1}{2} \left\| F_k - \widehat{F}_k \right\|^2, \tag{3.1}$$

Table 2
Results obtained by NSSGM, SSHBB and SSGM2 for experiment in Section 3.1.

P	NSSGM					SSHBB					SSGM2				
	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE
1	6	7	19	0.1792	9.92E−08	6	7	19	0.216	1E−07	6	7	19	0.1925	9.92E−08
	5	6	16	0.1854	4.53E−05	5	6	16	0.206	5E−05	6	7	19	0.2056	4.53E−05
	5	6	16	0.0672	2.11E−05	5	6	16	0.068	2E−05	6	7	18	0.0958	2.11E−05
2	22	100	67	0.0690	3.01E−23	22	100	67	0.083	2E−24	22	100	67	0.0686	7.06E−23
	27	116	82	1.1052	5.9E−21	–	–	–	–	–	68	276	205	1.4511	1.25E−21
	79	527	238	1.7029	5.68E−22	–	–	–	–	–	80	543	262	1.9173	1.88E−21
3	93	154	280	0.5856	6.93E−08	–	–	–	–	–	101	170	304	0.8468	6.94E−08
	100	157	301	1.6796	2.16E−08	–	–	–	–	–	125	307	376	1.8596	2.16E−08
	27	55	82	0.6532	1.99E−10	28	56	85	0.778	2E−13	27	55	82	0.7322	2E-10
4	102	202	307	0.6568	4.2E−10	74	138	223	0.531	4E−10	102	202	307	0.4758	4.2E−10
	38	72	115	0.6394	6.45E−11	33	56	100	2.156	7E−11	38	72	115	0.5193	6.45E−11
	28	50	85	1.4876	4.71E−11	23	36	70	1.338	5E−11	28	50	85	0.5996	4.71E−11
5	1	2	4	0.0153	0.5	1	2	4	0.018	0.5	1	2	4	0.031	0.5
	1	2	4	0.0122	0.5	1	2	4	0.02	0.5	1	2	4	0.0086	0.5
	1	2	4	0.0120	0.5	1	2	4	0.013	0.5	1	2	4	0.009	0.5
6	6	7	19	0.0260	1.3E−21	6	7	19	0.022	1E−21	6	7	19	0.0415	1.3E−21
	6	7	19	0.0402	4.54E−21	6	7	19	0.04	5E−21	6	7	19	0.0384	4.54E−21
	6	7	19	0.0522	7.77E−21	6	7	19	0.156	8E−21	6	7	19	0.061	7.77E−21
7	72	152	217	0.2600	7.55E−10	69	132	208	0.487	7E−10	72	152	217	0.2999	7.55E−10
	34	63	103	0.5783	1.48E−10	33	56	100	0.625	1E−10	34	63	103	0.4356	1.48E−10
	28	49	85	0.5871	8.67E−11	23	36	70	0.729	1E−10	28	49	85	0.5917	8.67E−11
8	6	8	19	0.0216	1.71E−23	6	8	19	0.061	7E−24	6	8	19	0.04	1.71E−23
	6	8	19	0.0544	1.71E−23	6	8	19	0.101	1E−23	6	8	19	0.0449	1.71E−23
	6	8	19	0.0990	1.7E−23	6	8	19	0.177	1E−23	6	8	19	0.0768	1.7E−23
9	4	5	13	0.0439	1500	4	5	13	0.03	1500	4	5	13	0.0519	1500
	4	5	13	0.1026	4500	4	5	13	0.05	4500	4	5	13	0.0551	4500
	4	5	13	0.1674	7500	4	5	13	0.072	7500	4	5	13	0.0781	7500
10	5	6	16	0.0094	3.23E−29	5	6	16	0.047	3E−29	5	6	16	0.032	3.23E−29
	5	6	16	0.0311	9.7E−29	5	6	16	0.039	1E−28	5	6	16	0.036	9.7E−29
	5	6	16	0.0389	1.62E−28	5	6	16	0.052	2E−28	5	6	16	0.0453	1.62E−28

at each instantaneous time $t_k \in [0, t_{\text{final}}]$, where t_{final} is the final task duration,

$$F_k = \left[\ell_1 \cos(x_1) + \ell_2 \cos(x_1 + x_2), \quad \ell_2 \sin(x_1) + \ell_2 \sin(x_1 + x_2)\right]^T,$$

$\ell_i, \ i = 1, 2$, denotes the length of the ith–rod and $\hat{F}_k$ represents the end effector controlled track. For the purpose of this experiment, $\hat{F}_k$ is controlled to track the following Lissajous curve

$$\hat{F}_k = \left[\frac{3}{2} + \frac{1}{5}\sin(t_k), \quad \frac{\sqrt{3}}{2} + \frac{1}{5}\sin\left(2t_k + \left(\frac{\pi}{2}\right)\right)\right]^T.$$

To successfully execute the tracking process, the following additional parameters are set: the initial joint states $x_0 = [0, \frac{\pi}{3}]$, $\ell_1 = \ell_2 = 1$ and the task duration, $t_{\text{final}} = 10$ s is subdivided into 200 equal parts.

Numerical results generated by the NSSGM are plotted in Fig. 1 where Fig. 1(a) describes the synthesized robot trajectories, Fig. 1(b) gives the end effector trajectory and desired path. Also, Fig. 1(c) and (d) present the tracking residual error on the x-axis and y-axis, respectively. Looking at Fig. 1, it is evident that the new NSSGM algorithm completes the task of synthesizing the robot trajectories, successfully. The residual error recorded by the NSSGM on both x-axis and y-axis is below 10^{-10}. This affirms the suitability of NSSGM to deal with real-world problems.

4. Conclusion

In this research article, we have proposed a new spectral gradient-based algorithm for solving NLS problems called NSSGM. The proposal is an improvement upon the recently developed algorithms by Awwal et al. [20] for solving the same class of problems. Unlike in [20], the formulation of our spectral parameter in the proposed algorithm was shown to be independent of any safeguarding scheme. We then show theoretically the global convergence of the proposed NSSGM algorithm under some mild standard assumptions. Moreover, we also verify the efficiency of the NSSGM algorithm by solving some benchmark test problems in the literature and comparing the results with the best performing algorithm in [20], i.e., SSHBB and SSGM2 [19]. As future work, the structured vector γ_{k-1} can be incorporated into conjugate gradient-like algorithms such as [34,35] and explore their respective efficiencies. Finally, we show the applicability of this algorithm in motion control of the robotic arm problem.

Table 3
Results obtained by NSSGM, SSHBB and SSGM2 for experiment in Section 3.1.

P	NSSGM					SSHBB					SSGM2				
	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE
11	8	12	25	0.0186	3.16E−08	8	12	25	0.03	3E−08	8	12	25	0.0362	3.16E−08
	8	12	25	0.0474	9.57E−09	8	12	25	0.095	1E−08	8	12	25	0.0518	9.57E−09
	3	4	10	0.0256	7.93E−08	4	5	13	0.056	2E−08	4	5	13	0.039	7.93E−08
12	564	1262	1693	2.7219	5.83E−13	592	1320	1777	2.756	5E−13	582	1302	1742	1.9758	5.83E−13
	882	1921	2647	11.4496	1.79E−13	714	1587	2143	9.885	2E−13	882	1921	2647	9.5491	1.79E−13
	717	1538	2152	10.7454	1.15E−13	625	1397	1876	13.73	1E−13	717	1538	2152	11.2624	1.15E−13
13	6	8	19	0.0294	3.58E−18	6	8	19	0.032	4E−18	6	8	19	0.058	3.58E−18
	6	8	19	0.0820	9.23E−18	6	8	19	0.037	9E−18	6	8	19	0.0572	9.23E−18
	6	8	19	0.0501	1.49E−17	6	8	19	0.054	1E−17	6	8	19	0.0726	1.49E−17
14	55	72	166	0.4069	9.77E−15	40	47	121	0.322	3E−16	55	72	166	0.664	9.77E−15
	40	47	121	1.8149	3.83E−15	44	51	133	2.323	2E−14	42	49	126	1.3627	3.83E−15
	47	57	142	2.6355	1.42E−14	40	47	121	2.992	6E−15	47	57	142	2.1729	1.42E−14
15	23	45	70	0.1381	8.98E−12	16	25	49	0.056	2E−16	23	45	70	0.111	8.98E−12
	23	45	70	0.1991	1.51E−11	16	25	49	0.176	7E−16	23	45	70	0.2202	1.51E−11
	23	45	70	0.5087	4.43E−11	16	25	49	0.245	1E−15	23	45	70	0.3459	4.43E−11
16	2	9	7	0.0109	6.92E−12	2	9	7	0.029	7E−12	2	9	7	0.0788	6.92E−12
	2	9	7	0.0234	2.08E−11	2	9	7	0.027	2E−11	2	9	7	0.0314	2.08E−11
	2	9	7	0.1528	3.46E−11	2	9	7	0.045	3E−11	2	9	7	0.044	3.46E−11
17	29	39	88	0.7268	2.53E−13	25	34	76	0.201	1E−10	29	39	88	0.252	2.53E−13
	29	39	88	0.8700	7.6E−13	24	32	73	0.641	4E−13	29	39	88	0.7458	7.6E−13
	29	39	88	0.7185	1.27E−12	24	32	73	0.957	6E−13	29	39	88	0.8161	1.27E−12
18	1	2	4	0.0117	0	1	2	4	0.308	0	1	2	4	0.0247	0
	1	2	4	0.0061	0	1	2	4	0.022	0	1	2	4	0.0095	0
	1	2	4	0.0094	0	1	2	4	0.042	0	1	2	4	0.0102	0
19	13	18	40	0.0251	3.26E−11	13	18	40	0.037	8E−16	13	18	40	0.0558	3.26E−11
	13	18	40	0.1883	9.86E−11	13	18	40	0.201	3E−15	13	18	40	0.0839	9.86E−11
	13	18	40	0.1775	1.65E−10	13	18	40	0.171	5E−15	13	18	40	0.149	1.65E−10
20	21	36	64	0.2394	9.7E−10	21	36	64	0.071	1E−09	21	36	64	0.1044	9.7E−10
	21	36	64	0.5041	9.7E−10	21	36	64	0.247	1E−09	21	36	64	0.3	9.7E−10
	21	36	64	0.8168	9.7E−10	21	36	64	0.542	1E−09	21	36	64	0.3651	9.7E−10

Table 4
Results obtained by NSSGM, SSHBB and SSGM2 for experiment in Section 3.1.

P	NSSGM					SSHBB					SSGM2				
	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE	#iter	#fval	#nmvp	#time	FVALUE
21	6	8	19	0.0557	3.06E−18	6	8	19	0.068	3E−18	6	8	19	0.0383	3.06E−18
	6	8	19	0.1007	8.76E−18	6	8	19	0.053	9E−18	6	8	19	0.0555	8.76E−18
	6	8	19	0.1433	1.45E−17	6	8	19	0.076	1E−17	6	8	19	0.0751	1.45E−17
22	20	35	61	0.0427	7.08E−10	20	35	61	0.05	7E−10	20	35	61	0.0781	7.08E−10
	21	36	64	0.1450	5.51E−10	21	36	64	0.301	6E−10	21	36	64	0.1885	5.51E−10
	21	36	64	0.2143	6.96E−10	21	36	64	0.41	7E−10	21	36	64	0.26	6.96E−10
23	23	38	70	0.0648	7.19E−07	23	38	70	0.07	1E−06	23	38	70	0.087	7.19E−07
	22	36	67	0.1897	405 000	15	38	46	0.147	405 000	22	36	67	0.2264	405 000
	24	39	73	0.5082	1 125 000	22	47	67	0.568	1E+06	24	39	73	0.3986	1 125 000
24	398	812	1195	3.0304	9.81E−09	230	489	691	1.711	9E−09	398	812	1195	2.6658	9.81E−09
	219	440	658	4.2371	1.33E−08	276	597	829	6.677	2E−08	219	440	658	3.587	1.33E−08
	289	599	868	7.4218	1.65E−08	303	641	910	12.29	5E−09	289	599	868	6.2345	1.65E−08
25	2	27	7	0.0210	1.39E−14	2	27	7	0.029	1E−14	2	27	7	0.0381	1.39E−14
	17	137	52	0.2718	1.84E−16	–	–	–	–	–	–	–	–	–	–
26	39	327	118	0.0239	1.73E−18	24	150	73	0.053	4E−18	22	43	67	0.0282	0.19894
27	1	11	4	0.0114	1010	1	11	4	0.028	1010	1	11	4	0.0139	1010
28	508	1219	1525	0.2006	4.28E−11	–	–	–	–	–	512	1269	1598	0.2104	8.36E−14
29	8	11	25	0.0051	1.2905	11	14	34	0.008	1.2905	11	14	34	0.0129	1.2905
30	1	2	4	0.0077	0	1	2	4	0.019	0	1	2	4	0.0132	0
31	14	30	43	0.0131	0.49999	13	29	40	0.013	0.5	15	38	52	0.0161	0.49999
32	29	67	88	0.0171	24.4921	41	100	124	0.035	24.492	30	72	92	0.0131	24.4921
33	29	47	88	0.0136	3.52E−15	28	45	85	0.023	5E−14	32	46	97	0.0145	3.9E−15

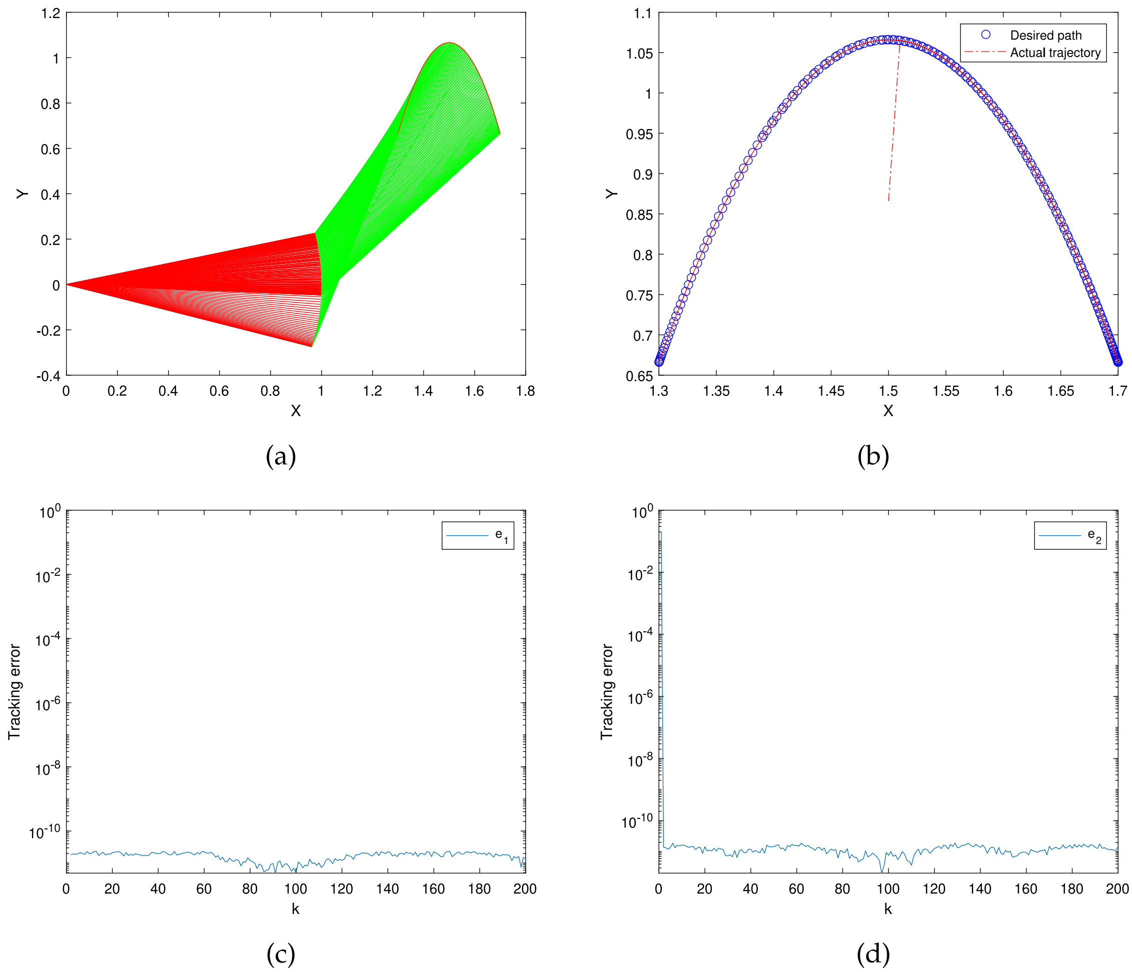

Fig. 1. Numerical results recorded by NSSGM method where: (a) Synthesized robot trajectories. (b) End effector trajectory and desired path. (c) Tracking residual error on the x-axis. (d) Tracking residual error on the y-axis.

Code availability

The MATLAB codes for the implementation of the proposed algorithm are available upon request.

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Data availability

No data was used for the research described in the article.

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