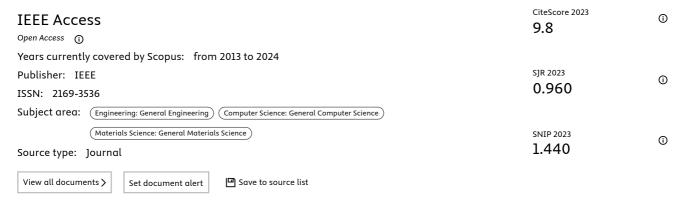


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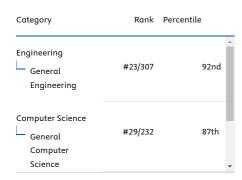
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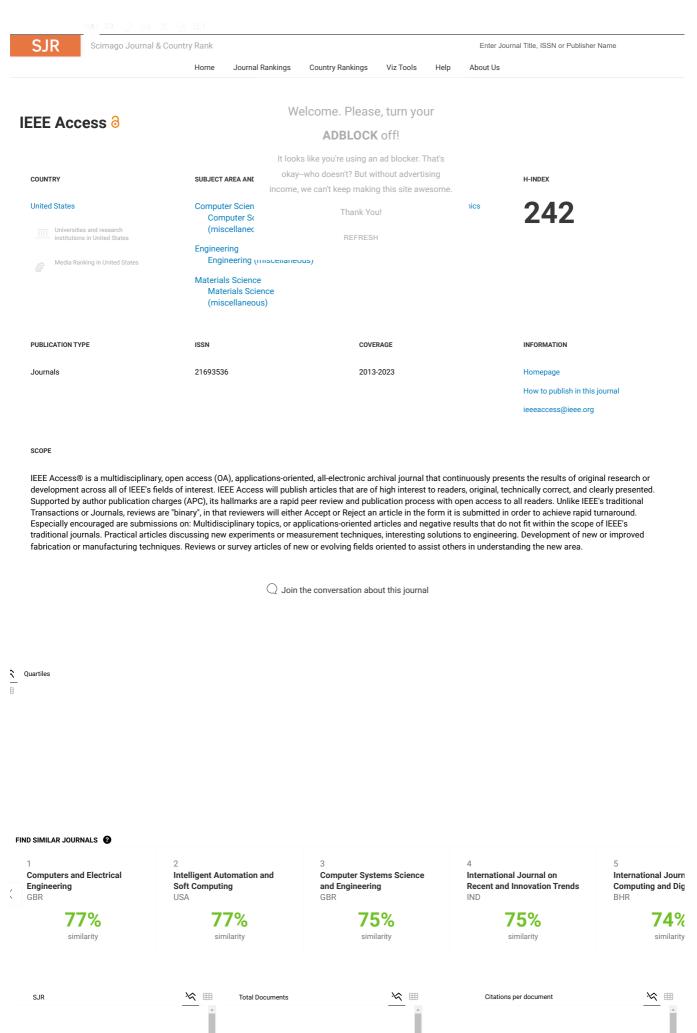
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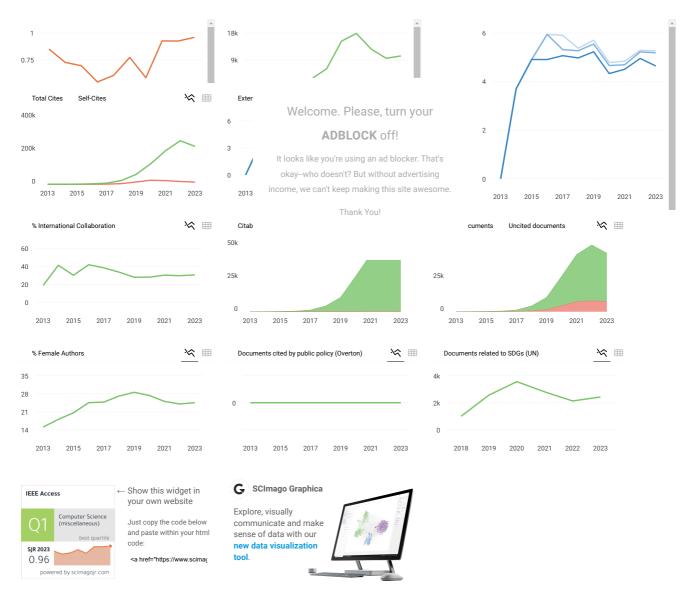
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15-Oct-2024

Dear Dr. Supharakonsakun:

Your manuscript entitled "Empirical Bayes Prediction for an Attribute Control Chart in Quality Monitoring" has been accepted for publication in IEEE *Access*. The comments of the reviewers who evaluated your manuscript are included at the foot of this letter. We ask that you make minor changes to your manuscript based on those comments, before uploading final files.

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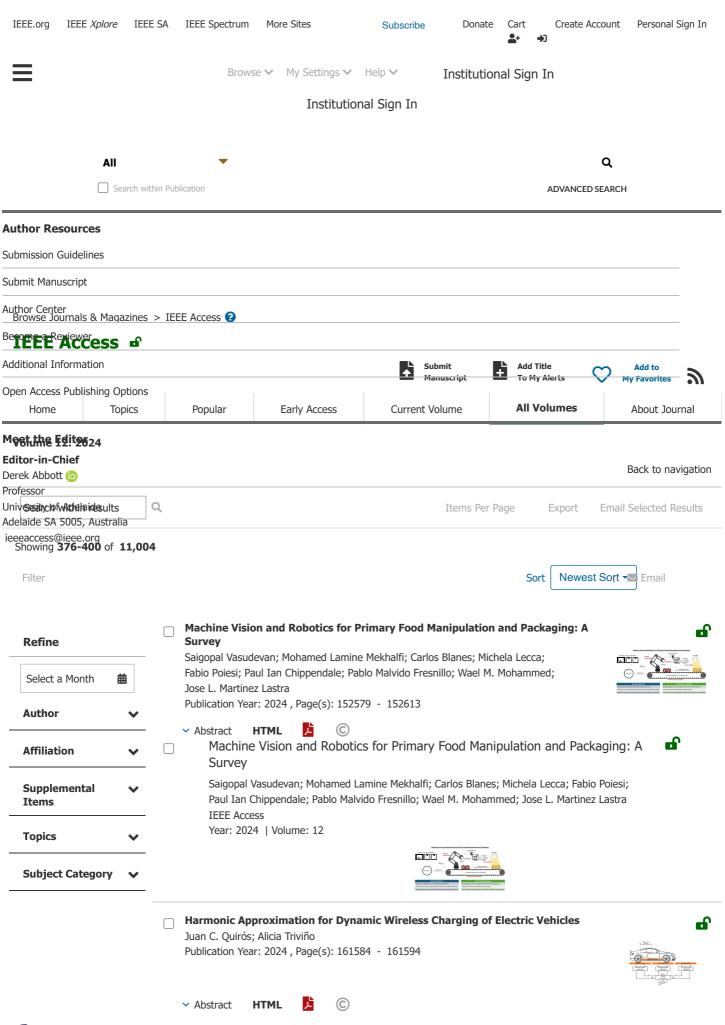
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Abstract





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Sanat Jain; Ashish Jain; Mahesh Jangid; Sucharitha Shetty

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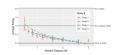


Assessing Quality Differences for 8K/UHD-2 and 4K/UHD-1 HDR Video Based on Viewing Distance



Dominik Keller; Rakesh Rao Ramachandra Rao; Alexander Raake

Publication Year: 2024, Page(s): 155024 - 155039



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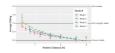
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Empirical Bayes Prediction for an Attribute Control Chart in Quality Monitoring

Yadpirun Supharakonsakun

Publication Year: 2024, Page(s): 160784 - 160793



Abstract



Empirical Bayes Prediction for an Attribute Control Chart in Quality Monitoring



Yadpirun Supharakonsakun

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Real-Time Dynamic Gesture Recognition Method Based on Gaze Guidance









Publication Year: 2024, Page(s): 161084 - 161095

Abstract

Real-Time Dynamic Gesture Recognition Method Based on Gaze Guidance



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Binbin Zhang; Weiqing Li; Zhiyong Su

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Empirical Bayes Prediction for an Attribute Control Chart in Quality Monitoring

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This work was supported by the Research and Develop Institute, Phetchabun Rajabhat University under Grant TSRI2567/67.

ABSTRACT A control chart is a valuable statistical tool used in production process control to ensure that products meet quality standards. The c-chart, a specific type of control chart, monitors the number of nonconformities or defects in a production process, thus maintaining product quality. The aim of this study is to introduce an improved c-chart for monitoring nonconformities via the Empirical Bayes approach. An exponential distribution, a special case of the single-parameter gamma distribution, is employed as suitable model for this analysis. To calculate the control limits, the posterior distribution and the predictive density are derived for the unconditional predictive density of the run length. The performance of the c-chart is evaluated via the average run length (ARL) and the standard deviation of the run length (SDRL), with a focus on Phase II analysis, where the chart is used for continuous monitoring of an in-control process. The proposed method's efficiency is compared with that of existing methods, demonstrating its superiority in achieving large ARL values for an in-control process, particularly when the parameter *c* is between 5 and 15. The effectiveness of the proposed method increases with increasing inspection unit size, highlighting its robustness and reliability. This enhanced performance highlights the advantages of the Empirical Bayes method in control chart applications, providing a practical and efficient tool for quality monitoring in various industrial processes.

INDEX TERMS Average run length, empirical Bayes, standard deviation of run length, predictive density, c-chart, Monte Carlo simulation, attribute control charts.

I. INTRODUCTION

In recent years, quality control methods have become increasingly vital for ensuring the efficiency and reliability of production processes across various manufacturing industries [1], [2], [3] and healthcare monitoring [4], [5], [6], [7]. Among these methods, the c-chart, a type of Shewhart control chart, has emerged as a prominent tool for monitoring the occurrence of nonconformities or defects in manufacturing processes. Developed for count data, the c-chart provides a systematic approach to tracking the number of nonconformities per inspection unit over time.

Despite its widespread adoption, the conventional approach to constructing c-charts often relies on assumptions that may not fully capture the complexity and variability

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inherent in manufacturing environments. Traditional methods for setting control limits typically involve specifying prior distributions based on expert judgment or historical data. This approach can lead to suboptimal results when faced with evolving process dynamics or limited data availability.

In response to these challenges, the Bayesian approach has been recognized as a powerful alternative to classical methods, particularly in term of statistical inference; it integrates historical information about parameters through prior distributions, making it beneficial in various contexts, including statistical process control (SPC) schemes. This approach is particularly useful in situations where prior knowledge can significantly inform parameter estimates [8], [9], [10], [11]. Moreover, Bayesian methods have been effectively applied in sequential sampling plans to estimate parameters and construct control limits based on posterior and predictive densities [12], [13], [14], [15], [16], [17]. The use of prior



distributions, where parameters are known or estimated independently from observed data, enhances the flexibility and accuracy of monitoring strategies.

Conversely, when the hyperparameters are unknown and estimated from the observed data, this method is referred to as the Empirical Bayes (EB) approach [18], [19], [20]. The Empirical Bayes approach has shown promising results in parameter estimation and classification research, demonstrating efficient performance across a wide range of applications [21], [22], [23], [24], [25], [26], including SPC [27], [28], [29], [30]. This method stands out by leveraging empirical data to refine prior parameter estimates, thereby improving the robustness and adaptability of control charts, such as the c-chart.

Recent advancements in the Empirical Bayes methodology have further enhanced its applicability in the SPC. For example, Empirical Bayes has been utilized to improve parameter estimation in sampling plans and other SPC-related applications [31], [32], [33], [34]. The incorporation of empirical data into the Bayesian framework allows for a more dynamic and responsive approach to quality control, addressing the limitations of traditional methods. In particular, the use of predictive densities based on non-informative Jeffreys prior has demonstrated superior performance compared with classical methods [35], [36], [37], [38]. This methodology improves the determination of average run length (ARL) by considering the run length distribution, which is crucial for assessing the performance of control charts [39], [40].

Moreover, recent research by Supharakonsakun [46] has extended the c-chart to the Bayesian methodology by the gamma distribution to establish control limits. Supharakonsakun's work compares the performance of the Bayesian approach with that of existing methods and shows that the Bayesian method offers larger ARLs and smaller false alarm rates (FARs), indicating improved effectiveness in process monitoring. However, challenges remain with large values of the λ parameter, suggesting that hyperparameter adjustment is necessary for optimal performance.

Similarly, Bayarri and Garcia-Donato [47] introduced a sequential, fully Bayesian approach to U-control charts, overcoming the limitations of the Poisson model and eliminating the need for a base period. Their work demonstrated that the Bayesian U-control chart is a powerful tool for process monitoring, highlighting the potential of Bayesian methods for improving control chart performance.

ARL measures the expected number of samples taken before the first out-of-control signal appears, thus reflecting the control chart's ability to monitor process stability. It is often associated with a geometric distribution, which corresponds to a nominal probability of 0.0027 that a point will exceed the 3-sigma control limits, resulting in an ARL of 370.4 for an in-control process [41]. A large ARL value is typically desirable for processes that are stable and in control.

The aim of this study aims to build upon the literature on both the c-chart and Empirical Bayes methodologies by exploring the application of Empirical Bayes techniques in enhancing the performance of the c-chart for quality control purposes. The proposed methodology focuses on using empirical data to estimate prior parameters, establishing control limits, and computing ARL values through predictive density procedures. In doing so, this study extends previous work by comparing frequentist and Bayesian methods with the Empirical Bayes approach, further contributing to the Phase II analysis of process monitoring and control.

II. CHARACTERISTIC OF C-CHART

In this research, the c-chart is employed to monitor the number of nonconformities in a production process. This type of control chart is specifically designed for count data, tracking the number of defects or nonconformities per inspection unit over time. The c-chart is extensively used in the manufacturing and service industries to oversee processes where defects may occur, such as in product quality control, machine performance monitoring, and evaluation of service errors.

To establish the c-chart via the collected data, the average number of nonconformities (\bar{c}) is calculated. The control limits of the c-chart are derived from this average. The upper control limit (UCL) and lower control limit (LCL) are determined via the following formulas:

$$UCL/LCL = \begin{cases} \bar{c} + 3\sqrt{\bar{c}} \\ \bar{c} + 3\sqrt{\bar{c}}. \end{cases}$$
 (1)

These limits help assess whether the process is in control (within limits) or out of control (outside limits). The inspection unit size must remain constant throughout the monitoring period to ensure accurate control limit calculation and interpretation periods to ensure accurate control limit calculation and interpretation.

III. RESEARCH METHODOLOGY

The c-chart, also known as a count chart, is a control chart employed in statistical process control to track the number of nonconformities (defects) within a fixed-size sample of products or processes over time. This chart is designed for count data, specifically monitoring the number of defects in a consistent inspection unit size, such as per item, batch, or area.

The c-chart is based on the statistical assumption that the number of nonconformities follows a Poisson distribution, which is suitable for infrequent events that occur independently within a fixed area or volume. When inspection units are chosen randomly at uniform time intervals, the count of nonconformities in the i^{th} inspection is expected to adhere to a Poisson distribution characterized by a specific parameter \bar{c} , represented by:

$$f(x_i|c) = \frac{e^{-c}\lambda^{x_i}}{x_i!}, x_i = 0, 1, 2, \dots, i = 1, \dots, m; c > 0.$$
(2)

In this study, an informative prior is utilized within the Bayesian approach. Assume that a random variable,



denoted X, follows an exponential distribution with parameter α , expressed as $X \sim Exp(\alpha)$. The probability density function of the exponential distribution is expressed as follows:

$$\pi(c) = \alpha e^{-\alpha c}; \alpha, c > 0. \tag{3}$$

Supharakonsakun and Jampachasri [24] proposed the Empirical Bayes estimator of λ by estimating the hyperparameter c via the maximum likelihood estimator (MLE) of the posterior marginal distribution (see [42]). The MLE for the hyperparameter c is obtained as follows:

$$\hat{\alpha}_{MLE} = \frac{m}{\sum_{i=1}^{m} x_i} = \frac{1}{\bar{x}}.$$
 (4)

The posterior distribution can be derived as follows:

$$h(c|\underline{X}) = \frac{L(c)\pi(c)}{\int L(c)\pi(c)dc},$$
 (5)

where $L\left(c\right)$ denotes the likelihood function of the Poisson probability mass function.

Thus, the posterior distribution can be expressed as follows:

$$h\left(c|\underline{X}\right) = \frac{\left(m+\hat{\alpha}\right)^{\sum\limits_{i=1}^{m}X_{i}+1}}{\Gamma\left(\sum\limits_{i=1}^{m}X_{i}+1\right)}e^{-\left(m+\hat{\alpha}\right)\lambda}\lambda^{\sum\limits_{i=1}^{m}X_{i}}.$$
 (6)

The posterior distribution of the parameter c follows a gamma distribution with parameters $\sum_{i=1}^{m} X_i + 1$ and $m + \hat{\alpha}$. It is expressed in the following form:

$$\pi_{j}(\lambda|data) = \frac{\left(n+\hat{\alpha}\right)^{\sum\limits_{i=1}^{m}x_{i}+1}}{\Gamma\left(\sum\limits_{i=1}^{n}x_{i}+1\right)}e^{-(n+\hat{\alpha})c}c^{\sum\limits_{i=1}^{m}x_{i}+\hat{\alpha}-1}_{ci}.$$
 (7)

The unconditional predictive density can be derived as follows [19]:

$$f\left(x_f|data\right) = \int_{0}^{\infty} f\left(x_f|c\right) \pi_j(\lambda|data) dc. \tag{8}$$

In this context, X_f represents the anticipated number of non-conformities in an upcoming inspection unit.

Here,

$$f(x_f|data) = \int_{0}^{\infty} \frac{e^{-c} c^{X_f} (m + \hat{\alpha})^{\sum_{i=1}^{m} X_i + 1} e^{-(m + \hat{\alpha})c} \lambda^{\sum_{i=1}^{m} X_i}}{X_f! \Gamma(\sum_{i=1}^{m} X_i + 1)} dc.$$
(9)

We have,

$$f(x_f|data) = \frac{(m+\hat{\alpha})^{\sum_{i=1}^{m} X_i + 1} \Gamma\left(\sum_{i=1}^{m} X_i + X_f + 1\right)}{X_f! \Gamma\left(\sum_{i=1}^{m} X_i + 1\right) (m+\hat{\alpha}+1)^{\sum_{i=1}^{m} X_i + X_f + 1}}.$$
(10)

The previously mentioned equation can be restated as a predictive density, illustrated as follows:

$$f\left(x_{f}|data\right) = \frac{\Gamma\left(\sum_{i=1}^{m} X_{i} + X_{f} + 1\right)}{\Gamma\left(\sum_{i=1}^{m} X_{i} + 1\right)\Gamma\left(X_{f} + 1\right)} \cdot \left(\frac{m + \hat{\alpha}}{m + \hat{\alpha} + 1}\right)^{\sum_{i=1}^{m} X_{i} + 1} \left(\frac{1}{m + \hat{\alpha} + 1}\right)^{X_{f}}.$$

$$(11)$$

Hence, the predictive density conforms to a negative binomial distribution characterized by parameters $\sum_{i=1}^{m} X_i + 1$ and

 $\frac{m+\hat{\alpha}}{m+\hat{\alpha}+1}$. This can be represented as

$$X_f \sim NB\left(\sum_{i=1}^m X_i + 1, \frac{m + \hat{\alpha}}{m + \hat{\alpha} + 1}\right). \tag{12}$$

In this research, the c-chart is employed to evaluate the performance of different methods for monitoring nonconformities. Specifically, the Empirical Bayes approach is compared with the classical and Bayesian methods, with a focus on the average run length (ARL) and standard deviation of the run length (SDRL). By conducting simulations and analyzing these performance metrics, the aim of this research is to demonstrate the effectiveness and robustness of the proposed Empirical Bayes c-chart in maintaining process control.

IV. SIMULATION RESULTS

This paper compares the unconditional average run lengths (ARLs) and unconditional standard deviation of run lengths (SDRLs) via classical methods, Bayesian methods with the Jeffreys prior [39], and the proposed method. The upper and lower control limits are computed for varying values of c and m. The Bayesian procedure is obtained through the predictive density.

The simulation study considers c = 1, 2, 3, 4, 5, 8, 10, 15, 20, 30, 40, and 50 and m = 5, 10, 15, 20, 25, 30, 50, 100, 200, and 500. The number of simulations is 20,000 iterations. The results of the proposed method are compared with those of the Raubenheimer and Merwe method and Chakraborti and Human method.

The results in Table 1 and Figure 1 show that for small inspection unit sizes (i.e., m = 5), the frequentist method provides the largest ARLs for most values of c (e.g., c = 1, 2, 3,



TABLE 1. Comparative unconditional ARL and SDRL performance for m = 5.

c	I	Ī.	I	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.523	1.961	2.461	1.896	2.495	1.922
2	6.599	6.079	6.371	5.849	6.440	5.919
3	16.621	16.113	15.907	15.399	16.151	15.643
4	39.451	38.948	37.894	37.390	38.700	38.197
5	87.983	87.481	87.201	86.699	88.846	88.344
8	457.528	457.028	495.864	495.363	508.543	508.042
10	403.320	402.820	394.117	393.617	396.740	396.240
15	322.985	322.484	284.340	283.840	286.119	285.619
20	303.445	302.944	258.346	257.845	261.055	260.555
30	270.425	269.925	226.336	225.835	225.724	225.224
40	258.646	258.146	213.615	213.115	213.802	213.301
50	258.564	258.063	209.998	209.497	213.302	212.802

Note: Bold indicates the maximum ARL for each method.

4, 10, 15, 20, 30, 40, and 50). This suggests that the classical method is more stable in these cases. However, the proposed Empirical Bayes method outperforms the classical approach for c=5 and 8, indicating its ability to provide better average run lengths in these specific scenarios. The SDRLs for all methods are consistently smaller than the ARLs, indicating that the variability in run lengths is well-controlled across all methods.

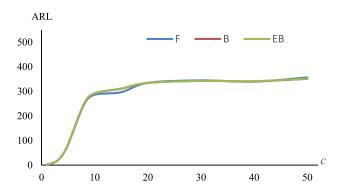


FIGURE 1. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 5.

Table 2 and Figure 2 show the performance for m=10. The frequentist approach continues to provide the largest ARLs for most values of c, particularly at c=1,2,3,4,5,15,20,30,40, and 50. However, the proposed method shows better performance for c=8 and 10, suggesting that it is more suitable for certain values of c even as the sample size increases. The SDRLs are slightly smaller than the ARLs, indicating that the variability is controlled.

Table 3 and Figure 3 present the performance for m=15. As the inspection unit size increases to m=15, the classical method continues to dominate for most values of c, providing the largest ARLs for c=1, 2, 3, 4, 20,

TABLE 2. Comparative unconditional ARL and SDRL performance for m = 10.

С	I	7	I	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.593	2.032	2.549	1.987	2.567	2.006
2	6.837	6.317	6.701	6.181	6.742	6.222
3	17.359	16.851	16.909	16.401	17.058	16.551
4	40.873	40.370	39.971	39.468	40.509	40.006
5	87.748	87.246	87.463	86.962	88.823	88.321
8	380.075	379.574	466.279	465.779	482.471	481.971
10	377.639	377.138	408.872	408.372	412.932	412.432
15	341.048	340.547	328.686	328.186	332.780	332.280
20	331.918	331.417	306.568	306.067	308.896	308.395
30	307.471	306.971	275.911	275.410	277.490	276.989
40	296.653	296.153	261.909	261.408	261.629	261.129
50	295.747	295.247	256.767	256.266	259.795	259.295

Note: Bold indicates the maximum ARL for each method.

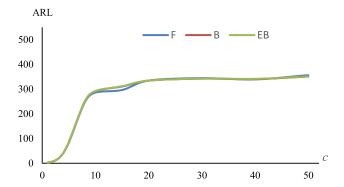


FIGURE 2. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 10.

TABLE 3. Comparative unconditional ARL and SDRL performance for m = 15.

<i>c</i>	I	F	I	3	E	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.618	2.058	2.588	2.027	2.600	2.040
2	6.877	6.357	6.789	6.269	6.814	6.294
3	17.469	16.962	17.110	16.602	17.199	16.691
4	41.470	40.967	40.538	40.035	40.998	40.495
5	87.749	87.247	87.373	86.872	88.979	88.478
8	333.879	333.379	421.245	420.744	434.286	433.786
10	355.781	355.281	402.691	402.190	403.637	403.136
15	338.532	338.031	344.104	343.604	342.652	342.152
20	333.037	332.537	320.628	320.127	322.077	321.576
30	322.993	322.492	296.287	295.786	298.564	298.064
40	316.832	316.332	288.396	287.896	289.298	288.798
50	311.756	311.256	280.851	280.350	281.538	281.037

Note: Bold indicates the maximum ARL for each method.

30, 40, and 50. However, the proposed method consistently outperforms the classical approach for intermediate values



of *c* such as 5, 8, 10, and 15. The results indicate that the Empirical Bayes method can be particularly effective for these moderate scenarios. The SDRLs remain smaller than the ARLs do, showing stable and consistent performance across all methods.

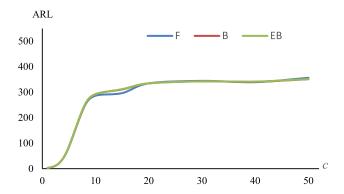


FIGURE 3. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 15.

TABLE 4. Comparative unconditional ARL and SDRL performance for m = 20.

	F		ī	3	EB	
<u> </u>						
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.629	2.069	2.606	2.045	2.614	2.054
2	6.930	6.411	6.851	6.331	6.875	6.355
3	17.648	17.141	17.340	16.832	17.458	16.951
4	41.850	41.347	41.156	40.653	41.513	41.010
5	87.438	86.937	87.767	87.266	88.412	87.911
8	320.636	320.135	391.105	390.605	400.954	400.454
10	354.353	353.853	392.213	391.712	397.724	397.224
15	337.875	337.374	343.017	342.516	346.345	345.844
20	338.400	337.900	330.766	330.266	331.824	331.323
30	329.345	328.845	309.481	308.981	312.139	311.639
40	323.651	323.151	298.056	297.555	300.750	300.250
50	321.599	321.099	295.052	294.551	297.839	297.339

Note: Bold indicates the maximum ARL for each method.

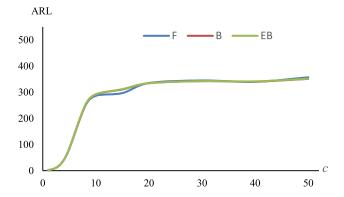


FIGURE 4. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 20.

Table 4 and Figure 4 show the performance for m=20, and the pattern remains largely consistent. The frequentist method provides the largest ARLs for extreme values of c (i.e., c=1, 2, 3, 4, 20, 30, 40,and 50), while the proposed method continues to outperform for intermediate values such as 5, 8, 10, and 15. The smaller SDRLs across all methods suggest that each method remains relatively stable, with low variability in performance for different sample sizes.

TABLE 5. Comparative unconditional ARL and SDRL performance for m = 25.

С	I	7	ŀ	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.633	2.073	2.616	2.056	2.623	2.063
2	6.935	6.416	6.878	6.358	6.891	6.372
3	17.683	17.176	17.420	16.912	17.504	16.996
4	42.110	41.607	41.408	40.905	41.726	41.223
5	88.076	87.575	88.326	87.825	89.116	88.615
8	301.673	301.173	357.517	357.017	367.692	367.192
10	344.584	344.083	379.129	378.629	387.569	387.068
15	340.014	339.513	345.630	345.130	350.446	349.946
20	337.992	337.492	333.388	332.887	335.973	335.473
30	330.354	329.853	315.095	314.594	317.694	317.194
40	327.309	326.809	308.244	307.743	309.001	308.500
50	326.897	326.397	304.271	303.770	303.898	303.398

Note: Bold indicates the maximum ARL for each method.

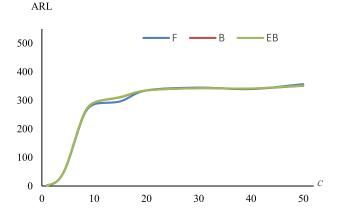


FIGURE 5. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 25.

Table 5 and Figure 5 show the results for m=25. The findings are consistent with those observed in previous scenarios. The frequentist method provides the largest ARLs for extreme values of c=1,2,3,4,20,30,40, and 50, whereas the proposed method performs better for intermediate values (c=5,8,10, and 15). This pattern confirms that the Empirical Bayes approach may offer a more balanced performance in cases where the process does not exhibit extreme levels of nonconformity. The SDRLs remain smaller than the ARLs do, showing controlled variability in the run lengths.



Table 6 and Figure 6 present the results for m=30. The proposed method continues to provide the largest ARLs for intermediate values (c=5, 8, 10, 15, and 20), while the frequentist method remains dominant for extreme cases (c=1, 2, 3, 4, 30, 40, and 50); this suggests that as the inspection unit size increases, the proposed method retains its relative advantage for moderate scenarios, whereas the frequentist approach maintains strong performance in more extreme cases.

TABLE 6. Comparative unconditional ARL and SDRL performance for m = 30.

С]	7	Ι	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.637	2.078	2.625	2.066	2.630	2.070
2	6.959	6.440	6.904	6.385	6.923	6.403
3	17.762	17.255	17.538	17.030	17.610	17.103
4	42.048	41.545	41.445	40.942	41.630	41.127
5	87.823	87.322	87.976	87.474	88.698	88.196
8	292.077	291.577	342.981	342.480	348.491	347.991
10	336.375	335.875	369.289	368.788	373.452	372.952
15	333.532	333.032	341.648	341.148	344.618	344.118
20	332.428	331.928	334.395	333.895	335.569	335.068
30	333.266	332.766	320.325	319.824	323.931	323.431
40	331.526	331.025	315.138	314.637	317.699	317.199
50	333.039	332.538	313.279	312.779	313.399	312.899

Note: Bold indicates the maximum ARL for each method.

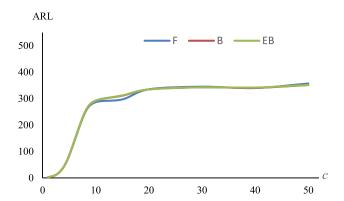


FIGURE 6. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 30.

Table 7 and Figure 7 display the results for m = 50. The proposed method remains effective for intermediate values of c, providing the largest ARLs for c = 5, 8, 10, 15, and 20, whereas the classical method continues to provide larger ARLs for extreme values. The SDRLs continue to be smaller than the ARLs across all methods, demonstrating that the variability in performance is kept under control.

Table 8 and Figure 8 present the results for m=100. The proposed method performs well for intermediate values of c=1,5,8,10,15, and 20, providing the largest ARLs in

these cases. The classical method still dominates for extreme values such as c=2, 3, 4, 30, 40, and 50. These results suggest that for larger inspection units, the proposed method maintains its strength in more balanced scenarios, whereas the classical method remains reliable in extreme cases.

Table 9 and Figure 9 show the results for m=200. The proposed method shows robust performance for intermediate values of c=1,4,5,8,10,20, and 40, providing the largest ARLs in these cases; this demonstrates the proposed to handle larger inspection units effectively. The Raubenheimer and Merwe method performs well for c=15, but overall, the classical method provides the largest ARLs for extreme cases. The SDRLs remain smaller than the ARLs, indicating stable performance.

Finally, Table 10 and Figure 10 show that for the largest inspection unit size of m=500, the proposed method continues to perform well, providing the largest ARLs for moderate values of c=1,4,5,8,15, and 40. The frequentist method remains dominant for very small and large values

TABLE 7. Comparative unconditional ARL and SDRL performance for m = 50.

С	I	7	I	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.639	2.080	2.633	2.074	2.636	2.077
2	6.987	6.467	6.956	6.437	6.961	6.442
3	17.874	17.367	17.719	17.211	17.772	17.264
4	42.440	41.937	42.120	41.617	42.272	41.769
5	88.692	88.191	88.601	88.100	89.172	88.670
8	275.537	275.037	305.024	304.524	308.782	308.281
10	322.610	322.109	344.030	343.530	348.806	348.306
15	332.920	332.419	339.781	339.281	341.314	340.813
20	334.633	334.132	337.154	336.653	338.263	337.763
30	336.721	336.221	331.844	331.344	332.113	331.612
40	337.013	336.512	328.052	327.552	328.280	327.780
50	338.275	337.775	326.485	325.984	327.199	326.699

Note: Bold indicates the maximum ARL for each method.

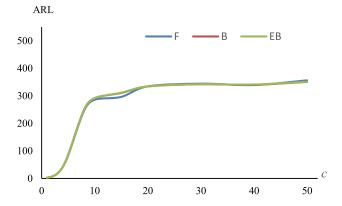


FIGURE 7. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 50.



TABLE 8. Comparative unconditional ARL and SDRL performance for m = 100.

С	I	7	I	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.640	2.080	2.639	2.080	2.641	2.081
2	7.032	6.513	6.995	6.476	7.002	6.482
3	17.994	17.487	17.872	17.365	17.876	17.369
4	42.607	42.104	42.435	41.932	42.544	42.041
5	88.163	87.662	88.641	88.140	89.154	88.653
8	260.599	260.099	277.658	277.157	279.641	279.141
10	308.645	308.144	325.186	324.685	327.430	326.930
15	324.453	323.953	332.131	331.631	335.977	335.477
20	333.668	333.167	335.598	335.097	336.423	335.922
30	339.305	338.804	337.579	337.079	338.611	338.110
40	341.465	340.965	338.116	337.616	339.181	338.681
50	345.013	344.513	339.606	339.105	338.987	338.487

Note: Bold indicates the maximum ARL for each method.

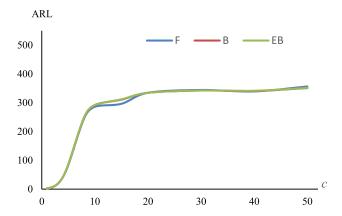


FIGURE 8. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m=100.

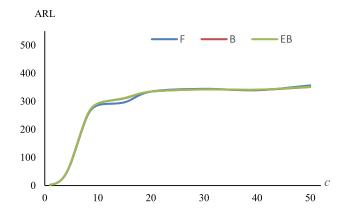


FIGURE 9. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 200.

 $c=2,\,3,\,20,\,30,\,$ and 50, whereas the Raubenheimer and Merwe method performs well for c=10.

TABLE 9. Comparative unconditional ARL and SDRL performance for m = 200.

С	I	7	I	3	Е	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.639	2.080	2.639	2.080	2.640	2.081
2	7.082	6.563	7.040	6.521	7.039	6.520
3	18.161	17.654	17.997	17.490	18.011	17.504
4	42.543	42.040	42.567	42.064	42.620	42.117
5	86.134	85.632	88.134	87.633	88.370	87.868
8	251.876	251.376	260.992	260.491	262.611	262.110
10	295.004	294.503	309.690	309.190	310.623	310.122
15	314.104	313.604	325.143	324.642	324.702	324.202
20	333.890	333.389	333.742	333.242	334.851	334.350
30	341.829	341.328	339.207	338.707	339.853	339.353
40	341.089	340.589	341.379	340.879	341.690	341.190
50	348.986	348.486	345.690	345.189	346.393	345.892

Note: Bold indicates the maximum ARL for each method.

TABLE 10. Comparative unconditional ARL and SDRL performance for m = 500.

	T	7	T	3	17	В
<u>c</u>	1		1	3	E	В
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1	2.639	2.079	2.639	2.080	2.639	2.080
2	7.132	6.613	7.097	6.578	7.099	6.580
3	18.447	17.940	18.247	17.740	18.253	17.746
4	42.564	42.061	42.544	42.041	42.638	42.135
5	83.401	82.900	85.504	85.002	85.542	85.041
8	247.050	246.550	249.871	249.371	250.478	249.978
10	286.322	285.822	292.276	291.776	291.856	291.356
15	296.652	296.152	310.884	310.384	311.839	311.338
20	334.853	334.352	334.780	334.280	334.449	333.949
30	344.675	344.175	342.611	342.110	342.669	342.169
40	339.667	339.167	340.913	340.413	341.575	341.075
50	356.779	356.279	351.161	350.661	351.543	351.043

Note: Bold indicates the maximum ARL for each method.

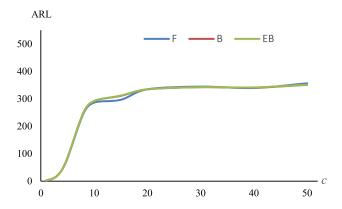


FIGURE 10. ARL curves of the frequentist, Bayesian and Empirical Bayes methods for m = 500.

The simulation results indicate that while the classical method tends to perform well for extreme values of c (very



TABLE 11. Comparison the performance methods for c = 20.

Method	LCL	UCL	CARL
Frequentist	6	32	200.7
Bayesian	8	35	267.50
Empirical Bayes	8	37	433.9642

low or very high nonconformity rates), the proposed Empirical Bayes approach consistently provides larger ARLs for intermediate values; this suggests that the proposed method is particularly well-suited to scenarios where the process exhibits moderate levels of nonconformity, offering a balanced approach to process monitoring. The SDRLs for all methods are smaller than the corresponding ARLs, indicating that the variability in run lengths is well-controlled and stable across all inspection unit sizes and process conditions.

The classical method's dominance in extreme cases is likely due to its ability to maintain consistent ARL performance under stable conditions, whereas the proposed method excels in more nuanced scenarios where empirical data can be used to adjust hyperparameters dynamically. The Raubenheimer and Merwe method sexhibits occasional effectiveness, particularly for certain values of \boldsymbol{c} , but is generally less competitive overall.

In summary, the proposed Empirical Bayes approach offers a robust framework for handling processes with moderate nonconformities, whereas the frequentist method provides reliable performance for more extreme conditions.

V. APPLICATION

Consider the example from Montgomery [45], Example 6-3 on Page 277, which has also been studied by Chakraborti, Human, Raubenheimer, and Merwe. This example examines the number of nonconformities observed in 26 successive samples of 100 printed circuit boards, with the inspection unit defined as 100 boards. Across the 26 samples, 516 nonconformities were found, resulting in an estimated $\bar{c} = 516/26 = 19.85$. Upon further investigation, units 6 and 24 were identified as out-of-control and subsequently removed. Revised control limits were then calculated using the remaining 24 samples, where m = 24 and $\sum_{i=1}^{m} x_i = 472$. The recalculated average number of nonconformities per inspection unit is $\bar{c} = 472/24 = 19.67$.

Using this example, we set m=24 and $\sum_{i=1}^{m} x_i = 472$. According to Human and Raubenheimer, the Jeffreys prior for c results in a gamma posterior distribution with parameters $\sum_{i=1}^{m} x_i + 0.5$ and m. In contrast, our proposed method uses an exponential prior via Empirical Bayes, resulting in a gamma posterior distribution with parameters $\sum_{i=1}^{m} x_i + 1$ and $m + \frac{1}{\bar{x}}$. For this example, the posterior distribution of c is Gamma (273, 24.0021).

To evaluate the performance of the c-chart for the observed value $\sum_{i=1}^{m} x_i = 472$, we investigate the unconditional ARL using the unconditional false alarm rate (FAR). Control

limits, the unconditional ARL is calculated via the frequentist and Raubenheimer and Merwe methods, and then compared with our proposed procedure using c=20 to calculate the CARL. The results are presented in Table 11.

Table 11 shows that the Empirical Bayes method provides a wider interval than the classical and Bayesian methods do, resulting in a larger CARL value. This is desirable for processes that are stable and in control.

VI. DISCUSSION

The Empirical Bayes approach in our study proved effective for designing a c-chart to monitor nonconformities, utilizing sample data to estimate prior parameters, thereby leading to a Bayesian predictive posterior distribution. This method involves the use of an exponential prior, which is a special case of the gamma distribution with a single parameter where the shape parameter equals 1. The selection of this prior was motivated by its simplicity and efficiency, as supported by relevant literature [39], [40]. The exponential prior simplifies the derivation of predictive densities for constructing control limits, contributing to the method's practical applicability.

In comparing our approach with previous methodologies, we observed that the Empirical Bayes method with an exponential prior yielded control limits, average run lengths (ARLs), and standard deviations of run length (SDRL) that were generally favorable, particularly for moderate to large inspection units. Our Monte Carlo simulations, comprising 20,000 iterations for ARL and SDRL calculations, revealed that the Empirical Bayes approach consistently provided high ARL values for parameters c=4 to 20, aligning well with the nominal value of 370.4. This large ARL is indicative of a lower false alarm rate, reinforcing the method's robustness.

However, the method's effectiveness was somewhat limited for small inspection units, where it presented slightly lower ARL values and greater variability. This suggests that while the Empirical Bayes approach is generally robust, there is room for improvement in scenarios involving small sample sizes or low nonconformity counts.

The literature on the selection of prior distributions in Bayesian inference supports the efficiency of the exponential prior in the Empirical Bayes context. For example, Gelman et al. [43] emphasized that the exponential prior provides a practical balance between simplicity and informativeness, making it suitable for a variety of applications where prior knowledge is either limited or straightforward. Additionally, Kass and Wasserman [44] highlighted the suitability of exponential priors in hierarchical models, noting their utility in empirical Bayesian frameworks due to their flexibility and ease of integration with posterior distributions.

For future work, exploring alternative priors such as other special cases of the gamma distribution or more informative priors based on additional historical data could enhance the method's adaptability and performance in diverse contexts. Such investigations would help refine the Empirical Bayes approach, ensuring its robustness across different parameter settings and inspection unit sizes.



VII. CONCLUSION AND RECOMMENDATIONS

The Empirical Bayes approach for designing a c-chart effectively utilizes sample data to estimate prior parameters, leading to a Bayesian predictive posterior distribution. Employing an exponential prior (a special case of the gamma distribution) simplifies the derivation of predictive densities, making it particularly useful for constructing control limits in quality monitoring.

Our analysis indicates that the Empirical Bayes method excels with moderate to large inspection units, providing optimal ARLs for parameters c = 4 to 20. Its performance diminishes slightly with smaller inspection units, suggesting a need for further adaptation in these cases. A key advantage of this method is its ability to avoid the arbitrary setting of hyperparameters, instead deriving them from sample data, thereby ensuring robust performance without the complications associated with inappropriate hyperparameter choices. Furthermore, our results indicate that smaller values of the hyperparameter α are associated with higher ARL values, demonstrating the benefit of selecting small α values for improved performance. The sensitivity of the hyperparameter is powerfully demonstrated in Table 12, which highlights the substantial impact of α on ARL performance for different sample sizes. Table 12 provides an unconditional ARL sensitivity analysis of α for inspection units m = 10, 30, and 50 given c = 10, revealing how smaller α values lead to significantly higher ARLs across all inspection units.

TABLE 12. Unconditional ARL sensitivity analysis of α for m=10,30 and 50 given c=10.

α	m=10	m=30	m=50
0.01	413.2467	371.7571	348.0638
0.10	395.5764	362.2525	342.0791
0.25	365.0740	348.8243	332.4718
0.50	319.8128	324.3323	316.2382
1	236.0555	282.1807	292.0034
2	123.5605	207.3667	240.1725
3	63.40634	157.3652	197.6921
4	34.94389	116.3317	166.0268
5	21.13531	89.94144	137.0188
10	4.66897	29.78343	61.66145

From our application results, the Empirical Bayes method provides the largest value of ARL compared with the classical and Bayesian methods. This finding indicates that the Empirical Bayes approach not only offers a robust framework for hyperparameter estimation but also enhances control chart performance, particularly for stable and in-control processes. This finding aligns with the conclusions of recent research by Supharakonsakun [46], which similarly highlighted the potential of Bayesian approaches in improving ARL performance in control charts for nonconformities

For future research, we recommend investigating the efficacy of alternative prior distributions within the Empirical Bayes framework. Exploring other forms of the gamma distribution or incorporating more informative priors based on extensive historical data could increase the method's flexibility and effectiveness. Additionally, further simulations and empirical studies could help identify optimal strategies for addressing scenarios with small sample sizes or low nonconformity counts, thereby broadening the applicability and robustness of the Empirical Bayes approach in various quality monitoring contexts. Recent research, such as that by Bayarri and Garcia-Donato [47], on U-control chart methodologies, provides valuable insights into alternative approaches and could serve as a foundation for future studies exploring advanced techniques in control chart design.

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