Engineering Letters

COUNTRY

Hong Kong

Universities and research institutions in Hong Kong

SUBJECT AREA AND CATEGORY

Engineering
└ Engineering (miscellaneous)

PUBLISHER

International Association of Engineers

H-INDEX

17

PUBLICATION TYPE

Journals

ISSN

1816093X, 18160948

COVERAGE

2009-2020

SCOPE

Information not localized

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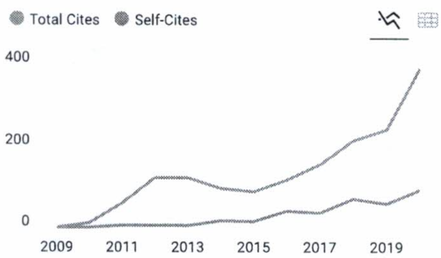

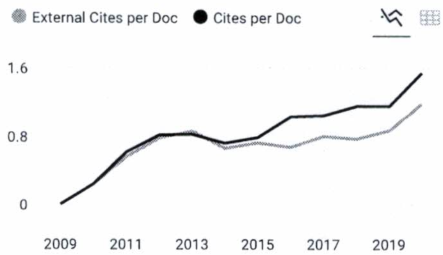

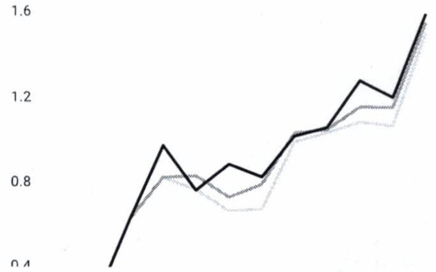

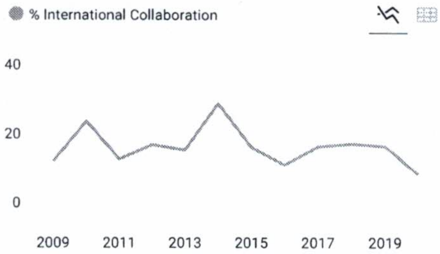

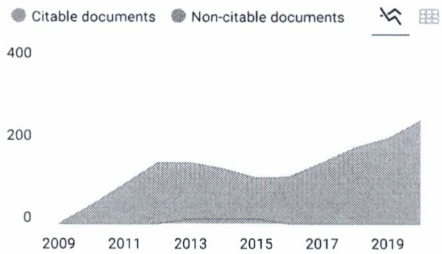

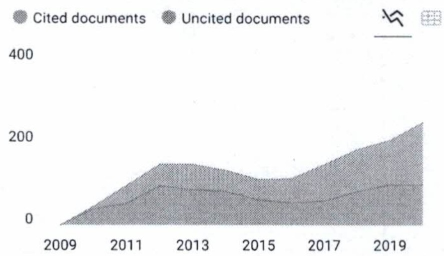

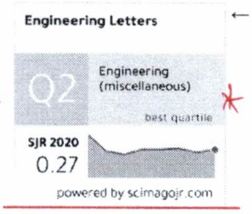

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Comparing the Effectiveness of Statistical Control Charts for Monitoring a Change in Process Mean

Yadpirun Supharakonsakun

Abstract— **The efficacy of control charts is usually evaluated by using the Average Run Length (ARL). A popular method for evaluating the exact ARL on a modified Exponentially Weighted Moving Average (EWMA) control chart is to use explicit formulas for the solution. In this study, explicit formulas were derived and implemented for the ARL on a modified EWMA control chart for a seasonal moving average of order q (SMA(q)) process when the white noise is exponentially distributed. In a comparison results between the explicit formula and the numerical integral equation method, are in good agreement. The explicit formulas were also used to assess the performance of the modified EWMA control chart was also compared to that of the traditional EWMA and Cumulative Sum control charts for a SMA(q) process. The superiority of the modified EWMA control chart was confirmed in that it was more sensitive to process mean changes than the other two for all smoothing parameter and shift size settings.**

Index Terms— **ARL, explicit formula, modified EWMA, seasonal moving average process**

I. INTRODUCTION

The control chart is a widely used tool for Statistical Process Control (SPC) that is traditionally implemented for monitoring and detecting shifts in the process mean or variance. This process monitoring tool is not solely used in industrial manufacturing but also many other areas such as environmental science [1]-[2], finance and economics [3]-[4], epidemiology and healthcare [5]-[6] among many others. The most popular control charts depending on their application include Shewhart [7], Cumulative Sum (CUSUM) [8], and Exponentially weighted moving average (EWMA) [9]. However, the said control charts are appropriate for processes with independent observations only, whereas in reality, this assumption is frequently violated. For a serially correlated process, the classical control charts must be modified to respond appropriately to shifts in the process parameter of interest. Recently, the modified EWMA control chart that overcomes the inertia problem in the standard EWMA control chart was proposed by Patel and Divecha [10]. It is an improved EWMA statistic that is the best predictor in the linear predictor class, and it is efficient of detecting small changes in the mean of the process, as well as abrupt shifts in autocorrelated process variables. Subsequently, Khan et al. [11] redesigned the control statistic of the modified EWMA control chart by multiplying a constant in the last term of its control statistic. This new control statistic performs more effectively than both the standard and modified EWMA control charts.

The performance of a control chart regarding its sensitivity in detecting small changes in a process parameter is usually measured by using the Average Run Length (ARL) classified into two states: ARL_0 is the expected number of in-control observations before the control charts falsely signals that the process is out-of-control whereas ARL_1 is the expected number of out-control observations before the control charts falsely signals that the process is out-of-control.

There are many methods used to evaluate the ARL values of a control chart for autocorrelated observations. For instance, Mastrangelo and Montgomery [12] proposed the ARL of an EWMA control chart for an autocorrelated observation by Monte Carlo simulation. Vanbrackle and Reynold [13] estimated the ARL on EWMA and CUSUM control charts for observations from a first-order autoregressive (AR(1)) model with additional random errors by using the integral equation and Markov Chain approaches. Similarly, Herdiani et al. [14] evaluated the ARL for an AR(1) process for a modified EWMA chart by using the Markov Chain method.

Other methods that are frequently used to evaluate the ARL for autocorrelated observations include Busababodin [15] , who presented explicit formulas for the ARL for a seasonal first-order moving average (MA(1)) process on a CUSUM control chart with exponential white noise. In addition, an ARIMA(p,d,q) model was derived the ARL by explicit formula for CUSUM control chart [16]. Petcharat [17] evaluated explicit formulas for the ARL on both EWMA and CUSUM control charts for a seasonal $AR(p)_L$ model with exponential white noise; the results indicate that the EWMA control chart is more effective than the CUSUM control chart for this scenario. Areepong and Sukparungsee [18] investigated the ARL for a CUSUM control chart of a seasonal AR integrated MA (SARIMA) process by deriving explicit formulas and confirmed their accuracy by using the Numerical Integral Equation (NIE) method; their findings show that the ARL obtained via explicit formulas was easy

Manuscript received March 4, 2021; revised April 13, 2021. This work was supported in part by Research and Development Institute, Phetchabun Rajabhat University, 67000, Thailand.

Y. Supharakonsakun is an Instructor of the Department of Mathematics, Faculty of Science and Technology, Phetchabun Rajabhat University, 67000, Thailand., (phone:665-671-7100; fax: 665-671-7110; e-mail: yadpirun.suph@ pcru.ac.th).

to derive, highly accurate, and computationally much faster than the NIE method. Petcharat [19] derived explicit formulas for the ARL of an EWMA control chart for a seasonal MA(q) (SMA(q)) process when the white noise is exponentially distributed and compared their performances on EWMA and CUSUM control charts; their results revealed that the performance of the explicit formulas on the EWMA control chart exceeded that on the CUSUM control chart for all magnitudes of shifts. Phanyaem [20] proposed explicit formulas for the ARL on a CUSUM control chart when the process is a seasonal first-order ARMA ($SARMA(1,1)_L$) process with exponential white noise. Peerajit et al. [21] investigated an approximation of the ARL on a CUSUM control chart for a long-memory process by using the NIE method when observations are from non-seasonal and seasonal AR fractionally integrated MA (ARFIMA) processes with exponential white noise. Suntornwat et al. [22] evaluated the analytical ARL for observations from a long-memory ARFIMA process on an EWMA control chart and also compared this with the same process running on a CUSUM control chart; their findings reveal that the EWMA control chart was more efficient than the CUSUM control chart under these circumstances. Sunthornwat and Areepong [23] derived explicit formulas for the ARL on a CUSUM control chart for seasonal and non-seasonal MA processes with exogenous variables; the accuracy of the ARL derived with explicit formulas was checked with derivations obtained by using the NIE method. Recently, the performance of the modified EWMA control chart are proposed for the first order autoregressive model. The efficiency of this procedure was extended to compare with the standard CUSUM and EWMA control charts determined by ARL. The finding indicate that the modified EWMA chart gives the best detection for small of the mean changes [24].

In the current study, explicit formulas of the ARL for a modified EWMA control chart when observations are from an SMA(q) process with exponential white noise were implemented and their accuracy was checked by comparison with the NIE method. In addition, the performance of the explicit formulas on a modified EWMA control chart was compared with the CUSUM and standard EWMA control charts.

II. The Characeristic of Modified EWMA Control Chart

The modified EWMA control chart was proposed in 2011 by Patel and Divecha which was modified from the original EWMA control chart by consider past observations similar to EWMA chart with an addition of the difference between latest changes and the previous ones in the process. It is effective for monitoring and detecting the changes of process in the observations which are autocorrelated or independent normal distribution. Afterwards, Khan et al. proposed the developed structure of the control statistic of the modified EWMA control chart. The newly redesigned control statistic is more efficient than original EWMA and modified EWMA control charts. The newly modified EWMA control chart statistic is defined as

$$Z_n = (1-\lambda)Z_{n-1} + \lambda Y_n + c(Y_n - Y_{n-1}),\ n = 1,2,3,..., \quad (1)$$

where λ is an weighted parameter which is $0<\lambda\le 1$, Y_n is a process and c is a constant.

The expected and variance of the control statistic are as following

$$E(Z_t) = \mu_0 \quad (2)$$

$$V(Z_t) = \left[\frac{\lambda + 2c\lambda + 2c^2}{2-\lambda}\right]\sigma^2 \quad (3)$$

where μ_0 is the target mean, σ is the process standard deviation.

Hence, the upper control limit (UCL), center line and lower control limit (LCL) of the modified EWMA control chart are defined as

$$UCL = \mu_0 + K\sigma\sqrt{\frac{\lambda + 2c\lambda + 2c^2}{2-\lambda}}, \quad (4)$$

$$CL = \mu_0, \quad (5)$$

$$UCL = \mu_0 - K\sigma\sqrt{\frac{\lambda + 2c\lambda + 2c^2}{2-\lambda}}, \quad (6)$$

where K is appropriate control width limit and the starting value $Z_0 = u$ and $X_0 = v$.

III. Derivation For ARL on Modified EWMA Chart

If Y_n is an observation of the seasonal moving average process denoted by $SMA(Q)_L$ which can be written as

$$Y_n = \mu + \varepsilon_n - \theta_1\varepsilon_{n-L} - \theta_2\varepsilon_{n-2L} - ... - \theta_Q\varepsilon_{n-QL}; n = 1,2,3,..., \quad (7)$$

where ε_t is exponential white noise. In general, the initial value μ is considered to be the process mean, θ is a moving average coefficient which $-1\le\theta\le 1$ and L is a period of time. Assuming that the process are in-control at time n where $0\le Y_n \le b$ providing that $L(u)$ denotes the ARL of modified EWMA chart. The integral equation is given by;

$$L(u) = 1 + \frac{1}{\lambda + c}\int_0^b L(k) f\frac{A}{(\lambda+c)}dk, \quad (8)$$

where

$$A = k - (1-\lambda)u + (\lambda\theta_1 + \theta_1)\varepsilon_{n-L} + (\lambda\theta_2 + \theta_2)\varepsilon_{n-2L} + ... + (\lambda\theta_Q + \theta_Q)\varepsilon_{n-QL} - (\lambda + c)\mu.$$

Therefore,

$$L(u) = 1 + \frac{1}{(\lambda + c)\beta}E, \quad (9)$$

TABLE I

COMPARISON THE ARL BY USING EXPLICIT FORMULA AND NIE METHOD FOR SMA(2)4 ON MODIFIED EWMA CHART WHEN GIVEN $\mu = 3, c = 1,\ \lambda = 0.05, 0.08$.

θ_i	δ	$\lambda = 0.05$			$\lambda = 0.08$		
		Explicit	NIE	ε_r	Explicit	NIE	ε_r
	0.00	500.053291	500.053280 (2.359)	2.199766E-06	500.076637	500.076621 (5.289)	3.199510E-06
	0.01	80.751091	80.751090 (2.344)	1.238373E-06	77.129979	77.129977 (2.594)	2.593025E-06
	0.03	30.157379	30.157379 (2.547)	0.000000E+00	28.700072	28.700071 (2.437)	3.484312E-06
$\theta_1 = 0.3$,	0.05	18.558185	18.558185 (2.406)	0.000000E+00	17.670904	17.670904 (2.422)	0.000000E+00
$\theta_2 = 0.5$	0.10	9.518045	9.518045 (2.391)	0.000000E+00	9.092412	9.092412 (2.390)	0.000000E+00
*b = 0.302413 b**=0.304222	0.20	4.943980	4.943980 (2.407)	0.000000E+00	4.755273	4.755273 (2.610)	0.000000E+00
	0.30	3.450022	3.450022 (2.500)	0.000000E+00	3.337733	3.337733 (2.391)	0.000000E+00
	0.40	2.726652	2.726652 (2.329)	0.000000E+00	2.650566	2.650566 (2.531)	0.000000E+00
	0.50	2.307688	2.307688 (2.437)	0.000000E+00	2.252075	2.252075 (2.610)	0.000000E+00
	1.00	1.537294	1.537294 (2.343)	0.000000E+00	1.517319	1.517319 (2.406)	0.000000E+00
	0.00	500.162976	500.162966 (2.344)	1.999348E-06	500.047705	500.047705 (2.313)	0.000000E+00
	0.01	55.339702	55.339702 (2.375)	0.000000E+00	52.626115	52.626115 (2.422)	0.000000E+00
	0.03	19.840735	19.840735 (2.719)	0.000000E+00	18.841970	18.841970 (2.391)	0.000000E+00
$\theta_1 = -0.3$,	0.05	12.077735	12.077735 (2.562)	0.000000E+00	11.488866	11.488866 (2.609	0.000000E+00
$\theta_2 = -0.5$	0.10	6.143360	6.143360 (3.000)	0.000000E+00	5.874766	5.874766 (2.454)	0.000000E+00
*b=0606770 **b=0.0608449	0.20	3.215636	3.215636 (2.609)	0.000000E+00	3.105177	3.105177 (2.500)	0.000000E+00
	0.30	2.292645	2.292645 (2.391)	0.000000E+00	2.230925	2.230925 (2.390)	0.000000E+00
	0.40	1.861496	1.861496 (2.484)	0.000000E+00	1.821913	1.821913 (2.297)	0.000000E+00
	0.50	1.620575	1.620575 (2.609)	0.000000E+00	1.593018	1.593018 (2.390)	0.000000E+00
	1.00	1.211269	1.211269 (2.344)	0.000000E+00	1.202990	1.202990 (2.343)	0.000000E+00

* for $\lambda = 0.05$, ** for $\lambda = 0.08$ and the parentheses of NIE are CPU time in seconds.

TABLE II

COMPARISON THE ARL BY USING EXPLICIT FORMULA AND NIE METHOD FOR SMA(2)4 ON MODIFIED EWMA CHART WHEN GIVEN $\mu = 3, c = 1,\ \lambda = 0.10, 0.12$.

θ_i	δ	$\lambda = 0.10$			$\lambda = 0.12$		
		Explicit	NIE	ε_r	Explicit	NIE	ε_r
	0.00	500.066102	500.066080 (2.265)	4.399418E-06	500.051741	500.051712 (2.281)	5.799400E-06
	0.01	74.898689	74.898678 (2.329)	1.468651E-05	72.800666	72.800664 (2.297)	2.747228E-06
	0.03	27.809692	27.809691 (2.390)	3.595869E-06	26.977618	26.977617 (2.359)	3.706776E-06
$\theta_1 = 0.3$,	0.05	17.129622	17.129622 (2.375)	0.000000E+00	16.624331	16.624331 (2.469)	0.000000E+00
$\theta_2 = 0.5$	0.10	8.832847	8.832847 (2.360)	0.000000E+00	8.590591	8.590590 (2.297)	0.000000E+00
*b = 0.305515 b**=0.3068731	0.20	4.640038	4.640037 (2.344)	2.155155E-05	4.532371	4.532371 (2.328)	0.000000E+00
	0.30	3.269056	3.269056 (2.360)	0.000000E+00	3.204814	3.204814 (2.437)	0.000000E+00
	0.40	2.603965	2.603965 (2.328)	0.000000E+00	2.560327	2.560327 (2.719)	0.000000E+00
	0.50	2.217971	2.217971 (2.375)	0.000000E+00	2.186005	2.186005 (2.531)	0.000000E+00
	1.00	1.505016	1.505016 (2.422)	0.000000E+00	1.493445	1.493445 (2.328)	0.000000E+00
	0.00	500.056680	500.056679 (2.266)	1.999773E-07	500.071346	500.071345 (2.312)	1.999715E-07
	0.01	50.963533	50.963533 (2.344)	0.000000E+00	49.405874	49.4058874 (2.281)	0.000000E+00
	0.03	18.233377	18.233377 (2.453)	0.000000E+00	17.665610	17.665610 (2.344)	0.000000E+00
$\theta_1 = -0.3$,	0.05	11.130310	11.130310 (2.344)	0.000000E+00	10.796002	10.796002 (2.500)	0.000000E+00
$\theta_2 = -0.5$	0.10	5.711168	5.711168 (2.312)	0.000000E+00	5.558597	5.558597 (2.359)	0.000000E+00
*b=0.0609776 **b=0.0611256	0.20	3.037777	3.037777 (2.329)	0.000000E+00	2.974834	2.974834 (2.359)	0.000000E+00
	0.30	2.193198	2.193198 (2.437)	0.000000E+00	2.157920	2.157920 (2.328)	0.000000E+00
	0.40	1.797681	1.797681 (2.343)	0.000000E+00	1.774996	1.774996 (2.344)	0.000000E+00
	0.50	1.576126	1.576126 (2.36)	0.000000E+00	1.560297	1.560297 (2.484)	0.000000E+00
	1.00	1.197892	1.197892 (2.375)	0.000000E+00	1.193098	1.193098 (2.344)	0.000000E+00

* for $\lambda = 0.10$, ** for $\lambda = 0.12$ and the parentheses of NIE are CPU time in seconds.

TABLE III

COMPARISON THE ARL BY USING EXPLICIT FORMULA AND NIE METHOD FOR SMA(3)$_4$ ON MODIFIED EWMA CHART WHEN GIVEN $\mu=3, c=1,\ \lambda=0.05, 0.08$.

θ_i	δ	$\lambda=0.05$ Explicit	NIE	ε_r	$\lambda=0.08$ Explicit	NIE	ε_r
	0.00	500.086550	500.086503 (2.469)	9.398373E-06	500.013051	500.012975 (2.547)	1.519960E-05
	0.01	99.218267	99.218261 (2.563)	6.047274E-06	95.251904	95.251897 (2.625)	7.348934E-06
	0.03	38.187020	38.187018 (2.562)	5.237382E-06	36.490083	36.490081 (2.593)	5.480941E-06
$\theta_1=0.3$,	0.05	23.697042	23.697041 (2.593)	4.219936E-06	22.643777	22.643776 (2.576)	4.416224E-06
$\theta_2=0.5$,	0.10	12.252087	12.252087 (2.562)	0.000000E+00	11.733794	11.733794 (2.594)	0.000000E+00
$\theta_3=0.7$	0.20	6.381118	6.381118 (2.563)	0.000000E+00	6.143727	6.143726 (2.579)	1.627676E-05
*b = 0.6138240 b**=0.6200940	0.30	4.433529	4.433529 (2.593)	0.000000E+00	4.288672	4.288672 (2.593)	0.000000E+00
	0.40	3.476595	3.476595 (2.610)	0.000000E+00	3.376353	3.376353 (2.610)	0.000000E+00
	0.50	2.914362	2.914362 (2.547)	0.000000E+00	2.839758	2.839758 (2.672)	0.000000E+00
	1.00	1.847197	1.847197 (2.610)	0.000000E+00	1.818688	1.818688 (2.579)	0.000000E+00
	0.00	500.174669	500.174669 (2.625)	0.000000E+00	500.010698	500.010697 (2.484)	1.999957E-07
	0.01	48.191052	48.191052 (2.578)	0.000000E+00	45.804936	45.804936 (2.687)	0.000000E+00
	0.03	17.086089	17.086089 (2.687)	0.000000E+00	16.231185	16.231185 (2.547)	0.000000E+00
$\theta_1=-0.3$,	0.05	10.374009	10.374009 (2.610)	0.000000E+00	9.875552	9.875552 (2.750)	0.000000E+00
$\theta_2=-0.5$,	0.10	5.274533	5.274533 (2.625)	0.000000E+00	5.051643	5.051643 (2.594)	0.000000E+00
$\theta_3=-0.7$	0.20	2.784147	2.784147 (2.625)	0.000000E+00	2.695408	2.695408 (2.593)	0.000000E+00
*b=0301074 **b=0.0301786	0.30	2.011788	2.011788 (2.594)	0.000000E+00	1.963593	1.963593 (2.719)	0.000000E+00
	0.40	1.657195	1.657195 (2.531)	0.000000E+00	1.627062	1.627062 (2.594)	0.000000E+00
	0.50	1.462485	1.462485 (2.593)	0.000000E+00	1.441982	1.441982 (2.578)	0.000000E+00
	1.00	1.144042	1.144042 (2.593)	0.000000E+00	1.138391	1.138391 (2.672)	0.000000E+00

* for $\lambda=0.05$, ** for $\lambda=0.08$ and the parentheses of NIE are CPU time in seconds.

TABLE IV

COMPARISON THE ARL BY USING EXPLICIT FORMULA AND NIE METHOD FOR SMA(3)$_4$ ON MODIFIED EWMA CHART WHEN GIVEN $\mu=3, c=1,\ \lambda=0.10, 0.12$.

θ_i	δ	$\lambda=0.10$ Explicit	NIE	ε_r	$\lambda=0.12$ Explicit	NIE	ε_r
	0.00	500.041419	500.041317 (2.515)	1.627676E-05	500.080836	500.080701 (2.516)	2.699564E-05
	0.01	92.805532	92.805525 (2.672)	7.542654E-06	90.501920	90.501912 (2.593)	8.839591E-06
	0.03	35.452528	35.452526 (2.625)	5.641347E-06	34.482211	34.482209 (2.594)	5.800092E-06
$\theta_1=0.3$,	0.05	22.000882	22.000881 (2.687)	4.545272E-06	21.400450	21.400449 (2.625)	4.672799E-06
$\theta_2=0.5$,	0.10	11.417624	11.417624 (2.610)	0.000000E+00	11.122477	11.122477 (2.797)	0.000000E+00
$\theta_3=0.7$	0.20	5.998753	5.998752 (2.641)	0.000000E+00	5.863307	5.863307 (2.735)	0.000000E+00
*b = 0.6244350 b**=0.6288990	0.30	4.200087	4.200087 (2.719)	0.000000E+00	4.117239	4.117239 (2.562)	0.000000E+00
	0.40	3.314973	3.314973 (2.688)	0.000000E+00	3.257514	3.257514 (2.609)	0.000000E+00
	0.50	2.794026	2.794025 (2.594)	0.000000E+00	2.751177	2.751177 (2.719)	0.000000E+00
	1.00	1.801140	1.801140 (2.515)	0.000000E+00	1.784651	1.784651 (2.718)	0.000000E+00
	0.00	500.232170	500.232170 (2.656)	0.000000E+00	500.188068	500.188068 (2.547)	0.000000E+00
	0.01	44.346438	44.346438 (2.593)	0.000000E+00	42.978927	42.978927 (2.610)	0.000000E+00
	0.03	15.710628	15.710628 (2.594)	0.000000E+00	15.224807	15.224807 (2.578)	0.000000E+00
$\theta_1=-0.3$,	0.05	9.572157	9.572157 (2.594)	0.000000E+00	9.289186	9.289186 (2.609)	0.000000E+00
$\theta_2=-0.5$,	0.10	4.915895	4.915895 (2.562)	0.000000E+00	4.789258	4.789258 (2.657)	0.000000E+00
$\theta_3=-0.7$	0.20	2.641259	2.641259 (2.594)	0.000000E+00	2.590675	2.590675 (2.610)	0.000000E+00
*b=0.0302367 **b=0.0303025	0.30	1.934130	1.934130 (2.531)	0.000000E+00	1.906572	1.906572 (2.688)	0.000000E+00
	0.40	1.608613	1.608613 (2.578)	0.000000E+00	1.591336	1.591336 (2.610)	0.000000E+00
	0.50	1.429413	1.429413 (2.672)	0.000000E+00	1.417630	1.417630 (2.610)	0.000000E+00
	1.00	1.134911	1.134911 (2.625)	0.000000E+00	1.131638	1.131638 (2.578)	0.000000E+00

* for $\lambda=0.10$, ** for $\lambda=0.12$ and the parentheses of NIE are CPU time in seconds.

TABLE V

COMPARISON THE PERFORMANCE OF MODIFIED EWMA, EWMA AND CUSUM CONTROL CHARTS BY ARL OF EXPLICIT FORMULA FOR SMA(2)$_3$ PROCESS GIVEN $ARL_0 = 500$, $\lambda = 0.05, 0.08, \theta_1 = 0.7, \theta_2 = 0.9$ AND c=1.

	$\lambda = 0.05$		$\lambda = 0.08$		CUSUM
δ	Modified	EWMA	Modified	EWMA	a=2
	(b=0.679504)	(h=5.67x10^{-7})	(b=0.687061)	(h=1.649x10^{-3})	(l=4.9947)
0.00	500.067144	500.108180	500.049154	500.169438	500.104209
0.01	102.621501	416.139736	98.621533	448.249678	462.518195
0.03	39.715038	291.221058	37.983919	362.249521	397.457474
0.05	24.681716	206.601080	23.603660	295.060186	343.594230
0.10	12.778421	92.607513	12.245878	182.372633	244.627159
0.20	6.658464	23.135199	6.413501	78.421997	135.923598
0.30	4.623722	7.612234	4.473789	38.431172	83.635273
0.40	3.621969	3.334461	3.517959	20.966007	55.873149
0.50	3.032271	1.942418	2.954706	12.525513	39.889863
1.00	1.908217	1.037616	1.878381	2.607975	13.185909

TABLE VI

COMPARISON THE PERFORMANCE OF MODIFIED EWMA, EWMA AND CUSUM CONTROL CHARTS BY ARL OF EXPLICIT FORMULA FOR SMA(2)$_4$ PROCESS GIVEN $ARL_0 = 500$, $\lambda = 0.05, 0.08, \theta_1 = -0.3, \theta_2 = -0.5$ AND c=1.

	$\lambda = 0.05$		$\lambda = 0.08$		CUSUM
δ	Modified	EWMA	Modified	EWMA	a=2
	(b=0.060677)	(h=6.25x10^{-6})	(b=0.0608449)	(h=0.02036)	(l=4.9945)
0.00	500.162976	500.068558	500.047705	500.019213	500.033518
0.01	55.339702	426.089100	52.626115	459.437083	462.454236
0.03	19.840735	312.210112	18.841970	422.817155	397.404891
0.05	12.077735	231.476098	11.488866	332.557387	343.550763
0.10	6.143360	114.934257	5.874766	229.161461	244.599546
0.20	3.215636	34.019576	3.105177	118.992912	135.911481
0.30	2.292645	12.504102	2.230925	68.119125	83.629425
0.40	1.861496	5.634068	1.821913	42.164886	55.870086
0.50	1.620575	3.097270	1.593018	27.822723	39.888135
1.00	1.211269	1.124885	1.202990	6.723415	13.834362

TABLE VII

COMPARISON THE PERFORMANCE OF MODIFIED EWMA, EWMA AND CUSUM CONTROL CHARTS BY ARL OF EXPLICIT FORMULA FOR SMA(3)$_3$ PROCESS GIVEN $ARL_0 = 500$, $\lambda = 0.10, 0.12, \theta_1 = 0.3, \theta_2 = 0.5, \theta_3 = 0.3$ AND c=1.

	$\lambda = 0.1$		$\lambda = 0.12$		CUSUM
δ	Modified	EWMA	Modified	EWMA	a=2
	(b=0.414508)	(h=0.04826)	(b=0.41673)	(h=0.5751)	(l=4.3217)
0.00	500.299660	500.142771	500.354660	500.057934	500.042078
0.01	81.661690	462.986922	79.466749	480.336369	465.937637
0.03	30.646518	398.441052	29.755547	444.514210	406.221837
0.05	18.930311	344.752193	18.385177	412.904853	356.029084
0.10	9.784868	245.370613	9.520760	348.480411	261.545182
0.20	5.138834	134.728221	5.019792	261.583223	152.814799
0.30	3.609922	80.759428	3.538094	207.001476	97.402213
0.40	2.863595	51.918904	2.814339	170.017921	66.547349
0.50	2.427786	35.335802	2.391407	143.280388	48.088732
1.00	1.611687	9.221855	1.598134	71.665377	16.333963

TABLE VIII
COMPARISON THE PERFORMANCE OF MODIFIED EWMA, EWMA AND CUSUM CONTROL CHARTS BY ARL OF EXPLICIT FORMULA FOR SMA(3)$_4$ PROCESS GIVEN $ARL_0 = 500$, $\lambda = 0.10, 0.12, \theta_1 = 0.3, \theta_2 = 0.5, \theta_3 = 0.7$ AND C=1.

δ	$\lambda = 0.1$ Modified (b=0.624435)	EWMA (h=0.02983)	$\lambda = 0.12$ Modified (b=0.628899)	EWMA (h=0.24523)	CUSUM a=2 (l=3.7185)
0.00	500.041419	500.059392	500.080836	500.057555	500.073684
0.01	92.805532	460.563152	90.501920	472.565436	467.835687
0.03	35.452528	392.459227	34.482211	423.312899	411.037530
0.05	22.000882	336.361565	21.400450	380.651656	362.906798
0.10	11.417624	234.121005	11.122477	296.389619	271.117161
0.20	5.998753	123.638810	5.863307	190.080051	162.815450
0.30	4.200087	71.733196	4.117239	129.471576	105.896798
0.40	3.314973	44.873873	3.257514	92.465825	73.385345
0.50	2.794026	29.855109	2.751177	68.605814	53.524758
1.00	1.801140	7.334717	1.784651	22.672400	18.245051

where

$$E = \int_0^b L(k) e^{\frac{k-(1-\lambda)u+(\lambda\theta_1+\theta_1)\varepsilon_{n-L}+(\lambda\theta_2+\theta_2)\varepsilon_{n-2L}+\ldots+(\lambda\theta_Q+\theta_Q)\varepsilon_{n-QL}}{\beta(\lambda+c)}+\frac{\mu}{\beta}} dk.$$

Suppose that

$$F(u) = e^{\left(\frac{(1-\lambda)u}{\beta(\lambda+1)}+\frac{\mu}{\beta}\frac{(\lambda\theta_1+\theta_1)\varepsilon_{n-L}+(\lambda\theta_2+\theta_2)\varepsilon_{n-2L}+\ldots+(\lambda\theta_Q+\theta_Q)\varepsilon_{n-QL}}{\beta(\lambda+c)}\right)}.$$

Then the function $L(u)$ can be defined as

$$L(u) = 1 + \frac{F(u)}{(\lambda+c)\beta}\int_0^b L(k)\cdot e^{-\frac{k}{\beta(\lambda+c)}} dk. \qquad (10)$$

The explicit formula of ARL for the seasonal moving average order q model with exponential white noise on the modified EWMA control chart is

$$ARL = 1 - \frac{\lambda e^{\left(\frac{(1-\lambda)u}{\beta(\lambda+c)}\right)}\left[e^{\left(-\frac{b}{\beta(\lambda+c)}\right)}-1\right]}{\lambda e^{\left(\frac{-\mu}{\beta}+\frac{(\lambda\theta_1+\theta_1)\varepsilon_{n-L}+(\lambda\theta_2+\theta_2)\varepsilon_{n-2L}+\ldots+(\lambda\theta_Q+\theta_Q)\varepsilon_{n-QL}}{\beta(\lambda+c)}\right)}+e^{\left(-\frac{\lambda b}{\beta(\lambda+c)}\right)}-1}, \qquad (11)$$

where the process is an in-control if $\beta = \beta_0 = 1$ and the process is an out-of control if $\beta = \beta_1 > 1$.

III. NUMERICAL RESULTS

The ARL of explicit formula is accredited by the Gauss-Legendre quadrature rule of NIE method with 500 nodes. The relative error of comparative study between the ARL obtained by explicit formula and NIE method is measured as;

$$\varepsilon_r = \frac{|ARL_{Explicit} - ARL_{NIE}|}{ARL_{Explicit}} \times 100\%, \qquad (12)$$

when ARL_{NIE} is the ARL of the numerical equation method was obtained by the integral equation in the (8) and can be examined by the Gauss-Legendre quadrature rule. It can be written as;

$$ARL_{NIE} = 1 + \frac{1}{\lambda+c}\sum_{j=1}^{m} w_j L(a_j) f\left(\frac{a_j - B}{\lambda+c} - \mu\right), \qquad (13)$$

where
$B = (1-\lambda)u + v + (\lambda\theta_1+\theta_1)\varepsilon_{n-L} + (\lambda\theta_2+\theta_2)\varepsilon_{n-2L} + \ldots$
$+ (\lambda\theta_Q+\theta_Q)\varepsilon_{n-QL}$.

Comparison of the ARLs derived using the explicit formulas and the NIE method on the modified EWMA, standard EWMA, and CUSUM control charts are reported in Tables I–IV. For the SMA(Q)$_L$ processes with exponential white noise given that ARL$_0$ = 500 (Tables V and VI), (θ_1, θ_2 =-0.3,-0.5) and (θ_1, θ_2 =0.7,0.9) with smoothing parameter λ = 0.05 and 0.08 for SMA(2)$_3$ and SMA(2)$_4$, respectively, while ($\theta_1, \theta_2, \theta_3$ =0.5,0.7,0.3) and ($\theta_1, \theta_2, \theta_3$ =0.3,0.5,0.7) with λ = 0.10 and 0.12 for *SMA(3)$_3$* and *SMA(3)$_4$*, respectively

Tables I to IV report the ARLs obtained via the explicit formulas and the NIE method for an SMA(Q)$_L$ process with exponential white noise of a modified EWMA control chart for λ = 0.05, 0.08, 0.10, and 0.12. The results show that the ARL values obtained with both methods are in excellent agreement. In addition, the explicit formulas took much less computational time than the NIE method.

Tables I to IV report the ARL values obtained via the explicit formulas and the NIE method for an *MA(Q)$_L$* process with exponential white noise on a modified EWMA control chart for λ = 0.05, 0.08, 0.10, and 0.12. The results present that the ARL values obtained with both methods are in excellent agreement. In addition, the explicit formulas took much less computational time than the NIE method.

Tables V–VIII summarize the ARLs for $MA(2)_3$ and $MA(2)_4$ process with $\lambda = 0.05$, 0.08 and $MA(3)_3$, $MA(3)_4$ process with $\lambda =$ 0.10, 0.12, respectively, given $ARL_0 = 500$ whereas the shift size = 0.01, 0.03, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50 and 0.10. The results show that the modified EWMA chart was much more sensitive for monitoring and detecting the process mean shifts than the classical CUSUM and EWMA control charts by showing a dramatic reduction in ARL. Thus, we conclude that the modified EWMA procedure performed more effectively than the traditional EWMA and CUSUM control charts for all magnitudes of shift size with smoothing parameter values of 0.08 and 0.12.

IV. Conclusion

Autocorrelated observations are frequently in the form of time series data in which serial correlation violates the traditional EWMA control chart regarding the observation independence assumption. In this study, the ARL of a modified EWMA control chart when the observations are from an SMA(q) model with exponential white noise was derived using explicit formulas. The accuracy of the analytical expressions was checked by comparison with the NIE method, the results of which were in good agreement with a relative error of less than 2.7×10^{-5}. Moreover, the explicit formulas were easy to derive and also reduced the computational time compared to the NIE method.

The smoothing parameter in the range of 0.05-0.25 is regularly recommended for detecting process mean shifts using the original EWMA procedure, while many researchers studied the appropriate of the ARL performance on a modified EWMA control chart and recommend the value of 0.1 as suitable for monitoring its performance. In this study, the smoothing parameter was varied as 0.05, 0.08, 0.1, and 0.12 in the comparison of the efficacies of the standard and modified EWMA, and CUSUM control charts. The findings illustrate that the modified EWMA procedure outperformed the traditional EWMA and CUSUM control charts in all cases except for a smoothing parameter value of 0.05.

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