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กลุ่มวิทยาศาสตร์และเทคโนโลยี

สารบัญ

หน้า

Determination model force time-dependent for serve tennis ball trajectory of motion
สร้างแบบจำลองของแรงที่เป็นฟังก์ชันของเวลาสำหรับผู้เสิร์ฟลูกเทนนิสที่มีการเคลื่อนที่แบบโพรเจ็กไทล์

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Abstract

In this paper, we can analyze the function force model that affects the behavior of a tennis ball displaced horizontally. We can evaluate the force function for initial trajectory velocity by using the theory of mathematics. The result show that the function force models which affects the behavior of the tennis ball to make the displacement horizontal farthest when decrease the applied force and initial velocity is exponential function force. the conclusion show that the model of exponential function make the tennis ball go farthest which compared with the both trigonometry function models.

Keyword: Displacement, Velocity, Force-time dependent for serve tennis ball

บทคัดย่อ

ในบทความนี้ เราสามารถวิเคราะห์แบบจำลองแรงเสิร์ฟของผู้เล่น ที่มีผลกระทบต่อลักษณะการกระจัดในแนวราบของลูกเทนนิส คำนวณหาสมการแรงเสิร์ฟโดยใช้ความเร็วเริ่มต้นผ่านทฤษฎีคณิตศาสตร์ และสร้างโมเดลฟังก์ชันของแรงที่มีผลต่อลักษณะการกระจัดในแนวราบของลูกเทนนิส เพื่อหาโมเดลฟังก์ชันของแรงที่ส่งผลต่อการเคลื่อนที่ของลูกเทนนิสที่มีระยะการกระจัดมากที่สุดเมื่อออกแรงน้อยที่สุด ซึ่งผลสรุปที่ได้ออกมาพบว่าเมื่อออกแรงในการตีและความเร็วในการโยนลูกขึ้นในอากาศเพียงเล็กน้อย โมเดลแรงในฟังก์ชันเอกซ์โพเนนเชียลมีผลทำให้ลูกเทนนิสเคลื่อนที่ได้ไกลที่สุด

คำสำคัญ: การกระจัด, ความเร็ว, แรงที่เป็นฟังก์ชันของเวลาสำหรับผู้เสิร์ฟลูกเทนนิส

Introduction

Tennis is a popular sport where a tennis ball is hit with racket over a net stretched across a court. In this time, tennis is one of the most popular global sports, so there are many researches that study about the behavior of tennis ball and racket. In 2000, R. Cross studied the dynamics of the interaction between the tennis ball that upon collision with the racket string in direction that determined by the coefficient of restoration. In 2008, R. Mehta studied the aerodynamics of tennis ball and wind tunnel mensuration endeavor. In 2010, S.N. Maitra studied about reaching after safely passing the ball over the net. In 2014, Gilliam J.P.de Carpenter explores the differences mapping algorithms for trajectories of the tennis ball.This study aims to contribute to resolving this problem by uniting theoretical foundations based on Newton's mechanics. In this project we present the method of evaluation of the horizontal and vertical displacement of the tennis ball. The scheme of the paper is as follows. In section 2 detailing with the ordinary differential equation. Next, we can establish models for vertical and horizontal displacement for serving the tennis ball and the result is glue given in section 3.

Theory of Mathematics and Method.

We show the theory of mathematics ordinary differential equation first-order non-homogenous linear equation. We give

$$a(x)\frac{dy}{dx}+b(x)\,y=g(x) \tag{1}$$

$a(x)$, $b(x)$ and $g(x)$ is a function of x , and $a(x)\neq 0$. We call eq. (1) Linear First-Order Differential Equation or Linear Equation. From eq. (1), bring $a(x)$ divided throughout the eq. (1), We have

$$\frac{dy}{dx}+\frac{b(x)}{a(x)}y=\frac{g(x)}{a(x)} \tag{2}$$

Set $\eta(x)=\dfrac{b(x)}{a(x)}$ and $f(x)=\dfrac{g(x)}{a(x)}$ we have

$$\frac{dy}{dx}+\eta(x)\,y=f(x) \tag{3}$$

Or

$$[\eta(x)\,y-f(x)]\,dx+dy=0 \tag{4}$$

Multiply integrating factor μ throughout the eq. (4) and find $\mu(x)$

$$\mu[\eta(x)\,y-f(x)]\,dx+\mu\,dy=0 \tag{5}$$

Make eq. (5) become Exact Differential Equations.

$$\frac{\partial}{\partial y}\mu[\eta(x)\,y-f(x)]=\frac{\partial\mu}{\partial x}$$

$$\therefore\ \mu\eta(x)=\frac{d\mu}{dx}$$

Due to μ is a function of x, When using the methods of variable separation.

$$\frac{d\mu}{\mu}=\eta(x)dx$$

And Integrate, We will get

$$\ln\mu=\int\eta(x)dx$$

$$\mu=e^{\int\eta(x)dx} \tag{6}$$

Take eq. (6) into eq. (3).

$$e^{\int \eta(x)dx}[\frac{dy}{dx}+\eta(x)\,\mathrm{y}]=e^{\int \eta(x)dx}f(x)$$

$$\frac{d}{dx}[\mathrm{y}\,e^{\int \eta(x)dx}]=e^{\int \eta(x)dx}f(x) \qquad (7)$$

And Integrated, We would get

$$ye^{\int \eta(x)dx}=\int e^{\int \eta(x)dx}f(x)dx+\mathrm{c} \qquad (8)$$

Then divided $e^{\int \mu(x)dx}$ over the eq. (8).We have the solution of the eq. (3)

$$y=e^{-\int \eta(x)dx}\left[\int e^{\int \eta(x)dx}f(x)dx+\mathrm{c}\right] \qquad (9)$$

There are three main forces acting upon a tennis ball in flight. They are in the order of importance: gravity $(m\vec{g})$, air-resistance force (F_d) and magnus force (F_L) as illustrated in Figure 1

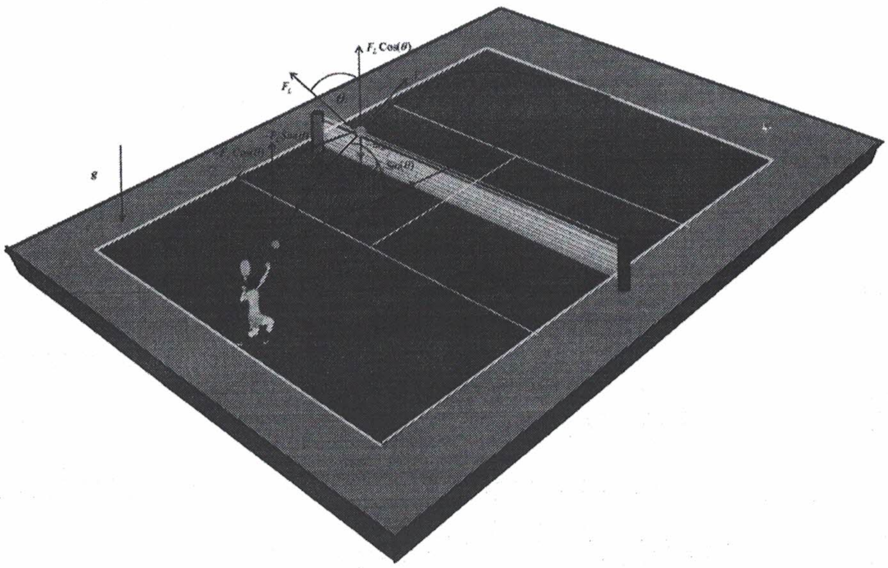

Figure 1: The three forces that act on a tennis ball in flight.

The gravitational force pulls the tennis ball vertically down towards the Earth with the corresponding gravitational acceleration $g=9.81m/s^2$. The drag resistance force and the magnus force are related to the tennis ball moving through the air. From figure 1, we can evaluate velocity in the horizontal (v_x) direction is

$$\sum F_x\hat{i}=m\frac{dv_x}{dt}\hat{i}$$

$$-\frac{F_L}{m}\sin(\theta)-\frac{F_d}{m}\cos(\theta)=\frac{dv_x}{dt}, \qquad (10)$$

Where $F_L=\kappa v_x$ is the magnus force (κ is called the magnus coefficient) and $F_d=\alpha v_x$ is the air-resistance force (α is called the drag-coefficient). So we would rewrite the eq. (10) to give

$$\frac{dv_x}{dt} + \beta v_x = 0, \quad (11)$$

Where $\beta = \left(\frac{\kappa}{m}\sin(\theta) + \frac{\alpha}{m}\cos(\theta)\right)$ is a function of θ. The eq.(11) maybe solved by direct integration by parts provided we know the initial conditions. Solving eq.(11) gives us the familiar result obtained in elementary mechanics as we show now. Assume at $t = 0$, the initial velocity is $v_x(t) = v_0\cos(\theta)$. The solution of eq.(11) is

$$v_x(t) = v_0\cos(\theta)e^{-\beta t} \quad (12)$$

Substituting $v_x(t) = dx(t)/dt$ into eq.(12) and assuming the initial condition that $x(0) = 0$ at $t = 0$, we get by direct integration

$$x(t) = \frac{v_0\cos(\theta)}{\beta}\left(1 - e^{-\beta t}\right) \quad (13)$$

eq.(13) is a horizontal displacement and we can rewrite eq.(2) to give

$$t(x) = -\frac{1}{\beta}\ln\left(1 - \frac{x(t)\beta}{v_0\cos(\theta)}\right) \quad (14)$$

eq.(14) is the time around the motion of the tennis ball. Then, considered from figure 1 again, we can also evaluate velocity in the vertical (v_y) direction is

$$\sum F_y\hat{j} = m\frac{dv_y}{dt}\hat{j}$$

$$\frac{dv_y}{dt} - \frac{F_L}{m}\cos(\theta) + \frac{F_d}{m}\sin(\theta) = -g \quad (15)$$

Where $F_L = \kappa v_y$ is the magnus force (κ is called the magnus coefficient) and $F_d = \alpha v_y$ is the air-resistance force (α is called the drag-coefficient). We would rewrite the eq.(15) to give

$$\frac{dv_y}{dt} + \mu v_y = -g \quad (16)$$

Where $\mu = \left(\frac{\alpha}{m}\sin(\theta) - \frac{\kappa}{m}\cos(\theta)\right)$ is a function of θ. The eq.(16) maybe solved by the first order differential equation. Provided the initial conditions. Solving eq.(16) for giving the familiar result obtained in the elementary mechanics, as we will present now. Let us assume that when $t = 0$, the initial velocity is $v_y(t) = v_0\sin(\theta)$. The solution of eq.(16) is

$$v_y(t) = \frac{-g}{\mu} + v_0\sin(\theta)e^{-\mu t} + \frac{g}{\mu}e^{-\mu t} \quad (17)$$

Putting $v_y(t) = dy(t)/dt$ into eq.(17) and assuming the initial condition that $y(0) = y_0$ at $t = 0$, so we also get this by direct integration too.

$$y(t)-y_0=\left(\frac{v_0\sin(\theta)(1-e^{-\mu t})-gt}{\mu}\right)+g\left(\frac{1-e^{-\mu t}}{\mu^2}\right) \quad (18)$$

eq. (18) is a vertical displacement.So we are finding the relation between the horizontal and vertical displacement by putting eq. (14) into eq. (18). That will give

$$y(x)=\left(\frac{v_0\sin(\theta)(1-e^{\frac{\mu}{\beta}\ln\left(1-\frac{x(t)\beta}{v_0\cos(\theta)}\right)})-g\left(\ln\left(1-\frac{x(t)\beta}{v_0\cos(\theta)}\right)\right)}{\mu}\right)+g\left(\frac{1-e^{-\frac{\mu}{\beta}\ln\left(1-\frac{x(t)\beta}{v_0\cos(\theta)}\right)}}{\mu^2}\right)+y_0 \quad (19)$$

So now, let us show the force calculation method for getting the initial velocity for trajectory.

From Figure 1, before the tennis ball can roll in the air. The tennis ball was hit by the racket. The collisions in this case is called elastic collisions in momentum. So we will study the three main forces for analyzing the motion of the tennis ball by using the momentum equation. eq. (20) the exponential function that affects the horizontal displacement of the tennis ball

$$F(t)=F_0e^{\beta_1 t} \quad (20)$$

We calculated eq. (20) by using direct integration. So we get

$$u_1=\frac{F_0(e^{\beta_1 t}-e^0)}{m\beta_1}+u_0 \quad (21)$$

eq. (21) is the initial velocity of trajectory. So we defined, F_0 is applied force and the velocity of throwing the tennis ball in the air is u_0

eq. (22) is the cosine function that also has an impact with the horizontal displacement of the tennis ball

$$F(t)=F_0te^{-rt}\cos(\theta) \quad (22)$$

We calculated eq. (22) by using direct integration by parts. So this give

$$u_1=\frac{-F_0\omega te^{-\gamma t}\sin(\omega t)+F_0r\omega te^{-\gamma t}\cos(\omega t)}{\omega^2+\gamma^2}-\frac{F_0e^{-rt}\cos(\omega t)}{\omega^2+\gamma^2}+\frac{2F_0\gamma\omega e^{-\gamma t}\sin(\omega t)-2F_0\gamma^2\omega e^{-\gamma t}\cos(\omega t)}{(\omega^2+\gamma^2)^2}+u_0 \quad (23)$$

eq. (23) is also the initial velocity of trajectory.

eq. (24) . the sine function that also has an impact with horizontal displacement of the tennis ball

$$F(t) = F_0 t e^{-\gamma t} \sin(\omega t) \tag{24}$$

We calculated eq. (24) by using direct integration by parts. So this give

$$u_1 = \frac{-F_0 t\omega e^{-\gamma t}\cos(\omega t) - F_0 t\gamma e^{-\gamma t}\sin(\omega t)}{\omega^2+\gamma^2} + \frac{F_0 e^{-\gamma t}\sin(\omega t)}{\omega^2+\gamma^2} - \frac{F_0\gamma\omega e^{-\gamma t}\sin(\omega t)}{(\omega^2+\gamma^2)^2} + \frac{F_0\gamma^2 e^{-\gamma t}\cos(\omega t)}{(\omega^2+\gamma^2)^2} + u_0 \tag{25}$$

eq. (25) is also the initial velocity of trajectory.

$$v_0 = (\frac{2m_1}{m_1+m_2})u_1 + (\frac{m_2-m_1}{m_1+m_2})u_2 \tag{26}$$

After we knew the initial velocity. We put them into eq. (26) , and get the initial velocity of the tennis racket. And put the initial velocity of the tennis racket into eq. (6) to show the behavior the horizontal displacement of tennis ball.

Result and Discussion.

eq. (6) is the solution of the first-order, linear equation non-homogeneous ordinary differential equation. In physics, it means vertical displacement depends on horizontal displacement. In this case, we study the force that affects the horizontal displacement behavior of the tennis ball by using the exponential function and trigonometry function for making the force models. So we get 3 cases for this study.

Case 1: we calculated from exponential function in figure 2 below

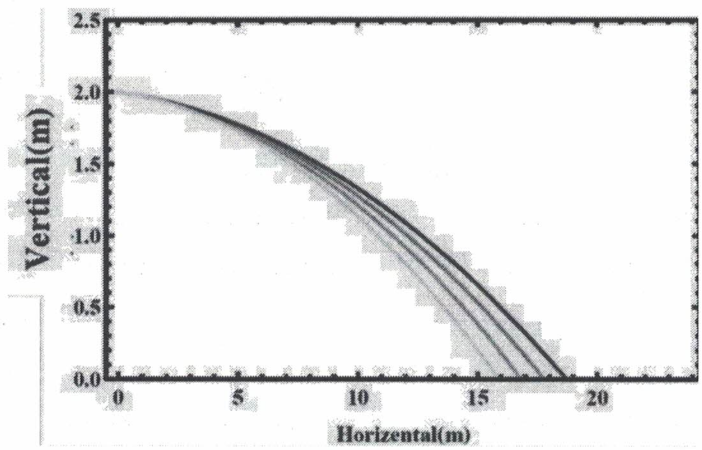

Figure 2: Representation of horizontal displacement behavior of the tennis ball.

The yellow line as velocity 26 m/s, pink line as velocity 28 m/s, green line as velocity 30 m/s and the purple line as velocity 32 m/s.From figure 2 we can analyze the displacement of the initial velocity from applied force of a tennis player's racket. The function force of time is exponential. The tennis ball served is elastic collisions. The final velocity of collision becomes an initial velocity of the tennis ball trajectory and put it in eq. (6) for finding the horizontal displacement behavior of the tennis ball.

Case 2: we calculated from Cosine function in figure 3 below

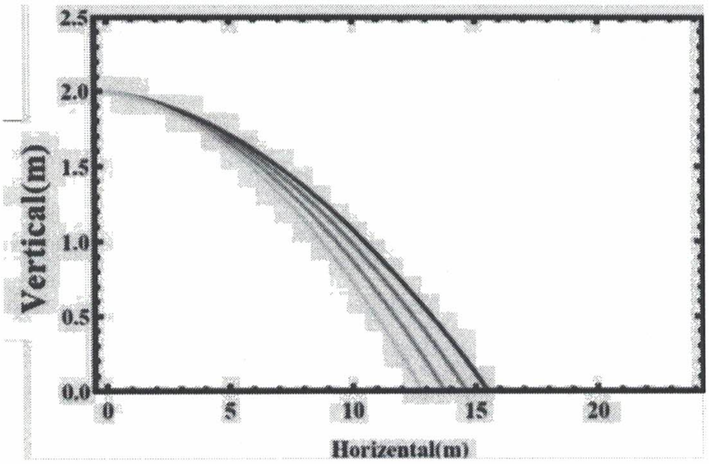

Figure 3: Plot of horizontal displacement behavior of the tennis ball.

The yellow line as velocity 26 m/s, pink line as velocity 28 m/s, green line as velocity 30 m/s and the purple line as velocity 32 m/s. From figure 2 we can analyze the displacement of the initial velocity from applied force of a tennis player's racket. The function force of time is cosine. The tennis ball served is elastic collisions. The final velocity of collision becomes an initial velocity of the tennis ball trajectory and put it in eq. (6) for finding the horizontal displacement behavior of the tennis ball.

Case 3: we calculated from exponential function as all line in figure 4 below

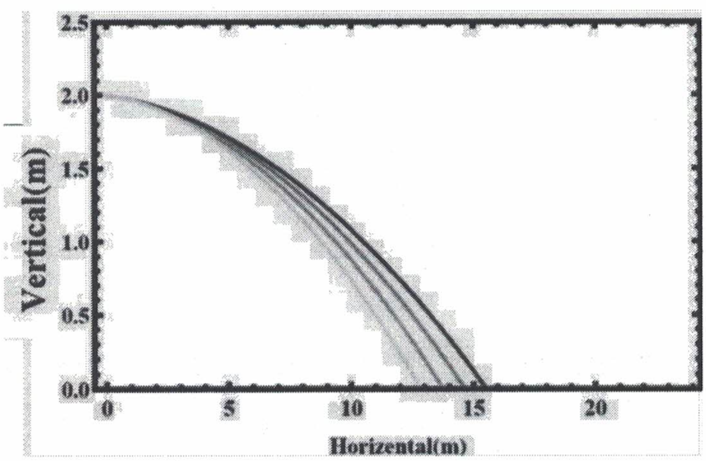

Figure 4:Illustration behavior of Displacement for horizontal of tennis ball.

The yellow line as velocity 26 m/s, pink line as velocity 28 m/s, green line as velocity 30 m/s and purple line as velocity 32 m/s.From figure 4 we can analyze the displacement of the initial velocity from applied force of a tennis player's racket. The tennis ball served is elastic collisions by using the function force of time is sine. The final velocity of collision becomes an initial velocity of the tennis ball trajectory and put it in eq. (6) for finding the horizontal displacement behavior of the tennis ball.

Next, comparison of the function force of time that affects horizontal displacement behavior of the tennis ball between exponential function and cosine function from figure 1 and 2. We will know that in the same strength and same height of the player. If it increases the initial velocity of hitting the racket and initial velocity of throwing tennis ball in the air, the horizontal displacement of exponential parameter will be farther than cosine parameter.Comparison the function force of time that affects the behavior of displacement for horizontal of tennis ball between exponential function and sine function from figure 1 and 3.When they are in the same strength and same height. If increase the initial velocity of hitting racket and initial velocity of throwing tennis ball in the air, the horizontal displacement of exponential parameter will be farther than sine parameter too.

So that, the exponential function suitable for the force function model for using in serving tennis ball. So if the player want toserving tennis ball to fall inside the baseline, they just decrease or increase only their applied force.

Conclusion

From numerical and result section, we can conclude that the function force models which affects the behavior of displacement for horizontal of tennis ball to make horizontal displacement of tennis ball farthest when decrease the applied force and initial velocity is exponential function force.

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Reference

Rod Cross, Effects of friction between the ball and strings in tennis, Sports Engineering, 2000, pp.85-97

Rabindra Mehta, Review of tennis ball aerodynamics, Sports technol, 2008, pp.7-16

S.N. Maitra, Projectile Motion Of A Tennis Ball Hitting The Court In Lawn Tennis Game, Bulletin of the Mathematical Society, 2010, pp.33-37

Gilliam J.P.de Carpentier, Analytical Ballistic Trajectories with Approximately Linear Drag, International Journal of Computer Games Technology, 2014, pp.1-13

Xinye Zhou and Xiuyun Chen, Study of Tennis Assisted Training System Based on Motion Tracking, Journal of applied Science and Engineering Innovation,2017,pp.80-85