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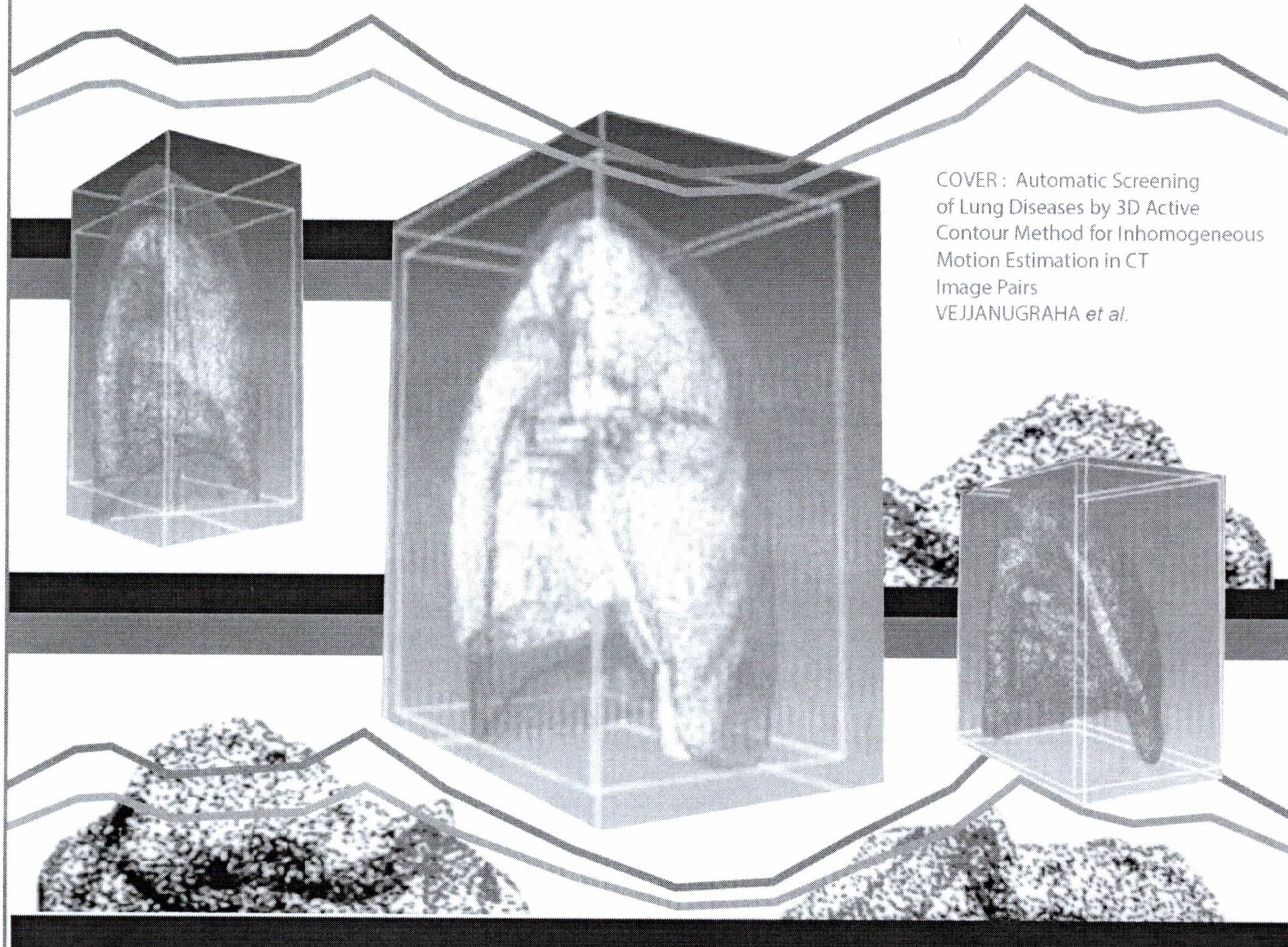

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Statistical Design for Monitoring Process Mean of a Modified EWMA Control Chart based on Autocorrelated Data

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Abstract

From the principles of statistical process control, the observations are assumed to be identically and independently normally distributed, although this assumption is frequently untrue in practice. Therefore, control charts have been developed for monitoring and detecting data which are autocorrelated. Recently, a modified exponentially weighted moving average (EWMA) control chart has been introduced that is a correction of the EWMA statistic and is very effective for detecting small and abrupt changes in independent normally distributed or autocorrelated observations. In this study, the performance of a modified EWMA chart is investigated by examining the 2 sides of the exact average run length based on an explicit formula when the observations are from a general-order moving average process with exponential white noise. A performance comparison of the EWMA and the modified EWMA control charts is also presented. In addition, the performance of the modified and EWMA control charts is contrasted using Dow Jones composite average from a real-life dataset. The findings suggest that the modified EWMA control chart is more sensitive than the EWMA control chart for almost every case of the studied smoothing parameter and constant values of the control chart.

Keywords: Autocorrelation, Two-sides of ARL, Explicit formula, Modified EWMA, Statistical control process

Introduction

Currently, the quality control of production processing is very important in industrial manufacturing. Statistical process control is a quality control tool which is widely used for monitoring and detecting changes in a process, and control charts are usually used for detecting shifts in the process mean. The 1st control chart, namely, the Shewhart chart, is useful for detecting large changes in a sequence of independent normally distributed observations with common variance coming from an individual process. Subsequently, many control charts have been presented, including the cumulative sum (CUSUM) control chart [1] and the EWMA control chart [2]. These procedures are widely useful for detecting statistical process control problems, and very insensitive to the normality assumption, which is important when using the control chart.

There are several situations which violate the assumption of independent normally distributed observations, the most common being autocorrelation. The considerable performance effectiveness for normally or non-normally distributed should be a desirable property to employ in process mean monitoring. A modified EWMA control chart, introduced by Patel and Divecha, is a procedure to detect and be free from the inertia problem. It has good performance for observations which are both autocorrelated and independently normally distributed. In addition, the advantage of the modified EWMA chart can be used to forecast the next period of observation. Therefore, it can indicate that protective action is needed before a process goes into an out-of-control state. The performance comparison of the modified EWMA and the EWMA control charts has also been studied. The results found that the modified EWMA control chart performed better that the existing one for all cases of smoothing parameter value [3]. This comparative performance for the control charts is determined by average run length (ARL).

ARL is one of the traditional measurements of a control chart's efficacy. There are 2 types: ARL_0 and ARL_1. The expectation of ARL_0 should be large, while that of ARL_1 should be as small as possible, since it comprises the expected number of observations before the control chart gives a false out-of-control state alarm. The smaller the ARL_1 is, the better the control chart performs.

Many studies to evaluate the ARL of control charts have investigated the normality and autocorrelation of observations. For instance, Champ and Rigdon used a Markov chain and an integral equation approach to approximate the ARL for CUSUM and EWMA charts; their results indicated that both methods attained the same approximations of ARL [4]. Yaschin proposed the ARL of a CUSUM control scheme for serially correlated observations [5], while Capizzi and Masarotto presented an adaptive EWMA control chart and compared it to standard Shewhart and EWMA charts by considering the ARL [6]. Vargas *et al.* evaluated the performance of CUSUM and EWMA control charts to detect small shifts in the process mean by determining the ARL [7], Apley and Lee proposed a method which was developed for designing residual-based EWMA control chart for uncertainly in the parameter model of autocorrelated process [8], and Mititelu *et al.* investigated explicit formulas for the ARL via an integral equation for an EWMA control chart with a Laplace distribution and a CUSUM control chart with a hyperexponential distribution [9]. Meanwhile, Areepong derived explicit formulas for observations that are binomially distributed on a moving average (MA) control chart [10], and Zhang *et al.* presented the monitoring of the coefficient of variation to further enhance the efficiency of the EWMA control scheme where the process mean and standard deviation are not constant [11]. Phanyaem *et al.* derived the ARL for autoregressive and moving average (ARMA) processes via explicit formula and numerical integral equation (NIE) method of EWMA control chart [12]. Phetcharat *et al.* investigated the derivation of the ARL for a moving average order q process with exponential white noise by explicit formula [13]. More recently, Sunthornwat *et al.* proposed the ARL of an EWMA chart for a long-memory autoregressive fractionally integrated MA process by analytical and numerical methods [14], and Herdiani *et al.* presented the performance of a modified EWMA chart via ARL where the observations are autocorrelated in the case of a 1^{st}-order autoregressive model [15]. Moreover, many researchers have investigated the performance of the modified EWMA control chart for various situations of observation and have also compared it with the existing control charts. Their results were determined by ARL and revealed that the modified EWMA control chart is more sensitive and effective to monitoring and detecting the changes in process mean [16-20]. Sunthornwat and Areepong proposed the derivation of explicit formulas of the ARL on a CUSUM control chart for seasonal and non-seasonal moving average processes with exogenous variables and exponential white noise [21].

The literature shows that various methods have been used to solve the ARL of a control chart and have provided similar values. An explicit formula is interesting to investigate, due to this method furnishing an exact solution, as well as saving on computational time. Therefore, in this study, the explicit formula of the 2 sides of the ARL on a modified EWMA control chart for observation in the general-order MA process with exponential white noise is evaluated, and a comparison of the performance of the modified EWMA chart with the standard one is also presented. Furthermore, the application of both control charts to analyze real observations is illustrated with real-life data.

Materials and methods

The modified EWMA control chart

In 2011, Patel and Divecha proposed and developed a modified EWMA control chart that is very effective for detecting small and abrupt changes in a process mean. The control statistic of this chart is a correction and is free from the inertia problem of the observations that are independent and normally distributed or autocorrelated. Afterward, Khan *et al.* [22] redesigned the structure of modified EWMA control statistic. The new control statistic was developed to be more effective with the classical of

EWMA and modified EWMA control charts. The control limits of the modified EWMA control chart with a weighted parameter λ, a constant k, the width of control limit L, the target mean μ, and standard deviation σ, are expressed as follows:

Upper control limit: $\mu + L\sigma\sqrt{\dfrac{\lambda + 2\lambda k + 2k^2}{2-\lambda}}$,

Center line: μ,

Lower control limit: $\mu - L\sigma\sqrt{\dfrac{\lambda + 2\lambda k + 2k^2}{2-\lambda}}$.

The modified EWMA statistic at time t is defined by the recursive:

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t + k(X_t - X_{t-1});\ t = 1,2,3,..., \tag{1}$$

where Z_t is the modified EWMA statistic with initial value $Z_0 = u$, and k is any constant and is independent of λ; Khan *et al.* showed that $k = -\lambda/2$ minimizes the variance of the modified control statistic. X_t is a sequence of autocorrelated observations. It is generated from the MA process.

The modified EWMA procedure performs well for observations which are autocorrelated or independently normally distributed due to the correction of the EWMA chart statistic.
The control limits of the EWMA control chart with a weighted parameter λ, width of control limit K, target mean μ, and standard deviation σ are comprised of:

Upper control limit: $\mu + K\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}$,

Center line: μ,

Lower control limit: $\mu - K\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}$,

The EWMA statistic at time t is defined by the recursive:

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t;\ t = 1,2,3,..., \tag{2}$$

where Z_t is the EWMA statistic with initial value $Z_0 = u$ and X_t is a sequence of autocorrelated observations.

The ARL of the modified EWMA chart based on a MA(q) model

A series of data points ordered in time is called a time series. It is regularly used to make forecasts of future value using time as an independent variable. The characteristic of a time series is found to be an autocorrelation. In the time series analysis, a moving average process is a normal approach for modelling a univariate time series variable by the white noise depending linearly on the current and various previous values of the white noise term. The general-order moving average process denoted as MA(q) process can be expressed as:

$$X_t = \mu + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - ... - \theta_q\varepsilon_{t-q}, \tag{3}$$

where μ and $|\theta_i| < 1;\ i = 1,2,3,...,q$ are the mean and coefficient of the MA process, respectively, and ε_t is white noise.

Therefore, the modified EWMA statistics can be written as:

$$Z_t = (1-\lambda)Z_{t-1} - X_{t-1} + (\lambda+k)\varepsilon_t - (\lambda\theta_1 + \theta_1)\varepsilon_{t-1} - (\lambda\theta_2 + \theta_2)\varepsilon_{t-2} - ... - (\lambda\theta_q + \theta_q)\varepsilon_{t-q} + (\lambda+k)\mu,$$

for $t = 1,2,3,\ldots$, $0 < \lambda \le 1$, where the initial value in the process mean $Z_0 = u$, $X_0 = v$, $\varepsilon_0 = s$, and the 2 sides of the control limit: $LCL = a$ and $UCL = b$. Thus:

$$Z_1 = (1-\lambda)u - v + (\lambda+k)\varepsilon_1 - (\lambda\theta_1+\theta_1)s - \ldots - (\lambda\theta_q+\theta_q)\varepsilon_{1-q} + (\lambda+k)\mu$$

Let $L(u)$ denote the ARL on the modified EWMA control chart for the q-order MA process. We can write the integral equation as:

$$L(u) = 1 + \int_{\frac{a-(1-\lambda)u+v+(\lambda\theta_1+\theta_1)s+\ldots+(\lambda\theta_q+\theta_q)\varepsilon_{t-q}-(\lambda+k)\mu}{\lambda+k}}^{\frac{b-(1-\lambda)u+v+(\lambda\theta_1+\theta_1)s+\ldots+(\lambda\theta_q+\theta_q)\varepsilon_{t-q}-(\lambda+k)\mu}{\lambda+k}} L\begin{bmatrix}(1-\lambda)u - v + (\lambda+k)y - (\lambda\theta_1+\theta_1)s - \ldots - (\lambda\theta_q+\theta_q)\varepsilon_{t-q} \\ +(\lambda+k)\mu\end{bmatrix} f(x)dx$$

To change the variable, the integral equation can be obtained by:

$$L(u) = 1 + \frac{1}{\lambda+k}\int_a^b L(j) f\left(\frac{j-(1-\lambda)u+v+(\lambda\theta_1+\theta_1)s+\ldots+(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{(\lambda+k)} - \mu\right)dj. \tag{4}$$

Since ε_t is a white noise process and is assumed to be exponentially distributed, the probability density function is $\frac{1}{\beta}e^{-\frac{y}{\beta}}; y \ge 0$ if Y is an identically independent distribution from an exponential distribution. Therefore:

$$L(u) = 1 + \frac{1}{\lambda+k}\int_a^b L(j)\cdot\frac{1}{\beta}\cdot e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}dj. \tag{5}$$

Let $E(u) = e^{\frac{((1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}$ and $F = \int_a^b L(j)\cdot e^{-\frac{j}{\beta(\lambda+k)}}dj$, thus:

$$L(u) = 1 + \frac{E(u)}{\beta(\lambda+k)}F. \tag{6}$$

Consider:

$$F = \int_a^b L(j)\cdot e^{-\frac{j}{\beta(\lambda+k)}}dj = \int_a^b\left[1+\frac{E(k)}{\beta(\lambda+k)}F\right]\cdot e^{-\frac{j}{\beta(\lambda+k)}}dj$$

$$F = \frac{-\beta(\lambda+k)\left[e^{-\frac{b}{\beta(\lambda+k)}} - e^{-\frac{a}{\beta(\lambda+k)}}\right]}{1+\frac{1}{\lambda}e^{\frac{v+(\lambda\theta_1+\theta_1)s+(\lambda\theta_2+\theta_2)\varepsilon_{t-2}+\ldots+(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}\left[e^{-\frac{\lambda b}{\beta(\lambda+k)}} - e^{-\frac{\lambda a}{\beta(\lambda+k)}}\right]}$$

Finally, the Fredholm integral equation of the 2nd kind is used to solve the ARL for the q-order MA process. The explicit formula of the 2 sides of the ARL on the modified EWMA control chart can be written as:

$$\mathrm{ARL} = 1 - \frac{\lambda e^{\frac{(1-\lambda)u}{\beta(\lambda+k)}}\left[e^{-\frac{b}{\beta(\lambda+k)}} - e^{-\frac{a}{\beta(\lambda+k)}}\right]}{\lambda e^{\frac{-\mu}{\beta}+\frac{v+(\lambda\theta_1+\theta_1)s+(\lambda\theta_2+\theta_2)\varepsilon_{t-2}+\ldots+(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}} + e^{-\frac{\lambda b}{\beta(\lambda+k)}} - e^{-\frac{\lambda a}{\beta(\lambda+k)}}}, \tag{7}$$

where $\beta = 1$ for an in-control process and $\beta > 1$ for an out-of-control process.

The explicit formula of 2 sides of the ARL on the EWMA control chart is derived likewise:

$$\mathrm{ARL} = 1 - \frac{\lambda e^{\frac{(1-\lambda)u}{\beta\lambda}}\left[e^{-\frac{h}{\beta\lambda}} - e^{-\frac{l}{\beta\lambda}}\right]}{\lambda e^{\frac{\mu}{\beta}\frac{\theta s}{\beta}} + e^{-\frac{h}{\beta}} - e^{-\frac{l}{\beta}}}, \tag{8}$$

where $h = UCL$ and $l = LCL$ of the control chart.

Existence and uniqueness of solution

In this section, the ARL on the modified EWMA control chart shows the existence and the uniqueness of the solution to the integral equation. The Banach's Fixed Point Theorem is used in this work for proving the existence and uniqueness of the solution of the ARL. It is important to prove that it is guaranteed to show the existence and uniqueness of the solution of the ARL. The unique existences of the solution of the integral equation of ARL are proved by the Banach's Fixed Point Theorem as follows:

Theorem 1 (Banach's Fixed Point) Let (X,δ) be a complete metric space and $T : X \to X$ be a contraction mapping with contraction $0 \le c < 1$ constant such that $\|T(L_1) - T(L_2)\| \le c\|L_1 - L_2\|$ for all L_1, $L_2 \in X$. Then there exists a unique $L(\cdot) \in X$ such that L_1, i.e., a unique fixed-point solution in X [23].

In this paper, let T be an operation in the class of all continuous functions, defined by:

$$T(L(u)) = 1 + \frac{1}{\lambda+k}\int_a^b L(j)\cdot\frac{1}{\beta}\cdot e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}dj. \tag{9}$$

According to the Banach's Fixed Point Theorem, if an operator T is a contraction, then the fixed-point solution of equation $T(L(u)) = L(u)$ has a unique solution.

Proof:

To show that $\|T(L_1) - T(L_2)\| \le c\|L_1 - L_2\|$ for all $L_1, L_2 \in X$ with $0 \le c < 1$.

Consider:

$$\|T(L_1) - T(L_2)\|_\infty = \sup_{u\in(a,b)}\left|\frac{e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}}{\beta(\lambda+k)}\int_a^b (L_1(j) - L_2(j))e^{-\frac{j}{\beta(\lambda+k)}}dj\right|$$

$$\le \sup_{u\in(a,b)}\left|\|L_1 - L_2\|_\infty e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}}\left(e^{\frac{-a}{\beta(\lambda+k)}} - e^{-\frac{-b}{\beta(\lambda+k)}}\right)\right|$$

$$= \left\| L_1 - L_2 \right\|_\infty \sup_{u \in (a,b)} \left| e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}} \right| \left| e^{\frac{-a}{\beta(\lambda+k)}} - e^{\frac{-b}{\beta(\lambda+k)}} \right|$$

$$\leq c \left\| L_1 - L_2 \right\|_\infty$$

where $c = \sup_{u \in (a,b)} \left| e^{\frac{-j+(1-\lambda)u-v-(\lambda\theta_1+\theta_1)s-(\lambda\theta_2+\theta_2)\varepsilon_{t-2}-\ldots-(\lambda\theta_q+\theta_q)\varepsilon_{t-q}}{\beta(\lambda+k)}+\frac{\mu}{\beta}} \right| \left| e^{\frac{-a}{\beta(\lambda+k)}} - e^{\frac{-b}{\beta(\lambda+k)}} \right|; \ 0 \leq c < 1$.

Hence, the existence and the uniqueness of the solution of ARL for the MA(q) process on the modified EWMA control chart are guaranteed by the Banach's Fixed Point Theorem. The unique solution satisfies the integral equation (7).

Results and discussion

Numerical results

The numerical results are evaluated by using (7) for the general-order MA process by arbitrary various parameters of the process and control chart given $ARL_0 = 370$ and 500.

Table 1 ARL of explicit formula of MA(2) process on modified EWMA charts for $\lambda = 0.08$, $\mu = 4$, $\theta_1 = -0.6$, and $\theta_2 = -0.8$, given $ARL_0 = 370$.

Shift size	$k = 0.05$ ($a = 0.201$, $b = 2.142$)	$k = 0.1$ ($a = 0.211$, $b = 2.5587$)	$k = 0.15$ ($a = 0.214$, $b = 2.6424$)
0.00	370.782782	370.406881	370.999034
0.01	349.601160	355.565492	359.422888
0.02	330.043955	341.615080	348.411994
0.05	279.693718	304.462070	318.410367
0.10	216.959561	255.093725	276.739244
0.20	139.944537	187.669725	215.286539
0.50	55.175928	95.889168	116.828470
1.00	23.185233	47.013894	52.815703
2.00	10.502035	19.214405	17.849241

Table 2 ARL of explicit formula of MA(2) process on modified EWMA charts for $\lambda = 0.1$, $\mu = 4$, $\theta_1 = -0.5$, and $\theta_2 = -0.7$, given $ARL_0 = 370$.

Shift size	$k = 0.05$ ($a = 0.226$, $b = 2.106$)	$k = 0.1$ ($a = 0.229$, $b = 2.2441$)	$k = 0.15$ ($a = 0.232$, $b = 2.29855$)
0.00	370.775185	370.249004	370.396938
0.01	352.931602	356.785017	358.499726
0.02	336.296535	344.046541	347.124678
0.05	292.680181	309.694743	315.831371
0.10	236.421237	262.940096	271.622730
0.20	163.304965	196.448857	204.994192
0.50	73.378308	98.631177	98.749432
1.00	33.041590	43.230366	39.039340
2.00	13.945032	15.262592	12.818871

Table 3 ARL of explicit formula of MA(3) process on modified EWMA charts for $\lambda = 0.08$, $\mu = 4$, $\theta_1 = -0.5$, $\theta_2 = -0.6$, and $\theta_3 = -0.7$, given $ARL_0 = 500$.

Shift size	***k* = 0.05** (*a* = **0.124,** *b* = **1.268)**	***k* = 0.1** (*a* = **0.129,** *b* = **1.5536)**	***k* = 0.15** (*a* = **0.132,** *b* = **1.62514)**
0.00	500.876911	500.769616	500.718926
0.01	472.119643	480.878873	485.879518
0.02	445.575305	462.187885	471.783937
0.05	377.273223	412.440996	433.483567
0.10	292.258343	346.430388	380.591215
0.20	188.065095	256.553685	303.409039
0.50	73.757005	135.398808	181.071734
1.00	30.847430	71.668426	92.840536
2.00	13.898632	31.397333	29.617440

Table 4 ARL of explicit formula of MA(3) process on modified EWMA charts for $\lambda = 0.1$, $\mu = 4$, $\theta_1 = -0.7$, $\theta_2 = -0.8$, and $\theta_3 = -0.9$, given $ARL_0 = 500$.

Shift size	***k* = 0.05** (*a* = **0.124,** *b* = **1.268)**	***k* = 0.1** (*a* = **0.129,** *b* = **1.5536)**	***k* = 0.15** (*a* = **0.132,** *b* = **1.62514)**
0.00	500.057004	500.012008	500.164168
0.01	476.104916	482.949740	487.237287
0.02	453.779386	466.826733	474.920368
0.05	395.265472	423.457734	441.260723
0.10	319.851462	364.750310	394.298822
0.20	221.999758	282.175943	324.764806
0.50	102.278086	163.873427	212.693232
1.00	49.493979	96.481174	127.653943
2.00	24.931807	47.211707	44.600348

Tables 1 - 4 report the 2 sides of ARL obtained by the explicit formulas of the modified EWMA control chart by fixing the values of a and seeking the values of b which correspond with the in-control processes $ARL_0 = 370$, 500. These are evaluated by (7) for a q-order MA process with exponential white noise for $\lambda =$ 0.08, 0.10 for various parameters of the 2nd and 3rd-order MA processes, respectively, given $ARL_0 =$ 370 and 500, where shift size is set as 0.01, 0.02, 0.05, 0.10, 0.20. 0.50, 1.00, or 2.00, and constant k is set as 0.05, 0.1, or 0.10.

Meanwhile, **Tables 5 - 8** contain the 2 sides of ARL obtained by the explicit formulas of the EWMA and modified EWMA control charts. These are evaluated by (8) and (7), respectively, for the 2nd and 3rd-order MA processes with exponential white noise for $\lambda = 0.08$, and 0.10, given $ARL_0 = 370$ and 500. The comparative results indicate that the performance of the modified EWMA chart is more efficient than the EWMA chart for almost all of the processes and weighted parameters and magnitudes of shift size. These are determined by the last row of each table, which is the relative mean index (*RMI*), defined as follows:

$$RMI = \frac{1}{m}\sum_{i=1}^{m}\left\{\frac{ARL_i - ARL_i^{smallest}}{ARL_i^{smallest}}\right\}, \tag{9}$$

where ARL_i denotes the ARLs of the EWMA and modified EWMA control charts for each shift size, and $ARL_i^{smallest}$ denotes the smallest of the ARLs for each sift size of 2 procedures.

Table 5 Comparison of the ARL of explicit formula of MA(2) process on EWMA and modified EWMA charts for $\lambda = 0.08$, $k = 0.05$, $\mu = 4$, $\theta_1 = -0.4$, and $\theta_2 = -0.5$, given $ARL_0 = 370$ and 500.

Shift size	$ARL_0 = 370$		$ARL_0 = 500$	
	EWMA (h = 0.518, l = 1.612)	**Mo EWMA (a = 0.197, b = 2.102)**	**EWMA (h = 0.315, l = 1.648)**	**Mo EWMA (a = 0.199, b = 2.217)**
0.00	370.657708	370.161235	500.780722	500.929569
0.01	350.428698	348.635540	472.065704	471.457095
0.02	331.701304	328.769938	445.557767	444.270211
0.05	283.252207	277.672836	337.346374	374.404080
0.10	222.358306	214.127833	292.476502	287.669070
0.20	146.647258	136.392773	188.673496	181.887371
0.50	61.861765	51.585875	75.931124	67.247849
1.00	30.031620	20.235502	35.396658	25.415077
2.00	19.832863	8.349178	23.103549	9.878156
RMI	**7.543068**	**0.000000**	**5.672496**	**4.632213**

Table 6 Comparison of the ARL of explicit formula of MA(2) process on EWMA and modified EWMA charts for $\lambda = 0.08$, $k = 0.1$, $\mu = 4$, $\theta_1 = -0.8$, and $\theta_2 = -0.6$, given $ARL_0 = 370$ and 500.

Shift size	$ARL_0 = 370$		$ARL_0 = 500$	
	EWMA (h = 0.477, l = 1.172)	**Mo EWMA (a = 0.213, b = 1.808)**	**EWMA (h = 0.453, l = 1.71)**	**Mo EWMA (a = 0.201, b = 1.851)**
0.00	370.720027	370.618598	500.453773	500.520543
0.01	350.271895	347.699099	471.448782	469.321084
0.02	331.347305	326.639145	444.681781	440.666227
0.05	282.416788	272.910814	375.847280	367.625308
0.10	220.990933	207.110748	290.311792	278.320361
0.20	144.786610	128.574500	185.942144	172.013579
0.50	59.897260	46.635099	73.213142	61.650111
1.00	28.360042	18.084682	33.076011	23.462372
2.00	18.598868	7.639766	20.965806	9.550522
RMI	**10.171981**	**0.000000**	**9.109647**	**0.000000**

Table 7 Comparison of the ARL of explicit formula of MA(3) process on EWMA and modified EWMA charts for $\lambda = 0.1$, $k = 0.1$, $\mu = 4$, $\theta_1 = -0.3$, $\theta_2 = -0.5$, and $\theta_3 = -0.4$, given $ARL_0 = 370$ and 500.

Shift size	$ARL_0 = 370$		$ARL_0 = 500$	
	EWMA (h = 0.242, l = 1.638)	**Mo EWMA (a = 0.212, b = 1.706)**	**EWMA (h = 0.302, l = 2.0)**	**Mo EWMA (a = 0.21, b = 1.786)**
0.00	370.861105	370.431869	500.640043	500.100764
0.01	350.538224	350.266510	471.828031	472.697885
0.02	331.725881	331.591345	445.234854	447.326544
0.05	283.066879	283.237077	376.826060	381.666022
0.10	221.933982	222.334711	291.767059	299.041462
0.20	145.981497	146.226221	187.866825	195.938592
0.50	61.061758	59.497873	75.433862	78.762174
1.00	29.233205	24.896232	35.624402	32.172067
2.00	18.868838	10.310609	26.266464	12.639240
RMI	**1.858167**	**0.101956**	**2.134945**	**3.309499**

The results in **Table 5** show the ARLs of the MA(2) process for $\lambda = 0.08$ on the EWMA and modified EWMA control charts. It is revealed that, when $ARL_0 = 370$, the performance of the modified EWMA control chart is better than the EWMA control chart for all magnitudes of shift size with the zero *RMI*, whereas for $ARL_0 = 500$, the small *RMI* of the modified EWMA control chart was 4.632213, while the *RMI* of the EWMA control chart was 5.672496.

Tables 6 and **8** report the ARLs of MA(2) and MA(3) processes for $\lambda = 0.10$. The results indicate that the modified EWMA control chart was more effective than the original one for all cases of ARL_0 with the zero *RMI*.

Table 7 contains the ARLs for MA(3) process of the EWMA and modified EWMA control charts when $\lambda = 0.08$. The results appeared to be that, when $ARL_0 = 370$, the performance of the modified EWMA control chart is better than the EWMA control chart for all magnitudes of shift size with the small *RMI* of 0.101956, whereas the *RMI* of the EWMA control chart was 1.858167. When $ARL_0 = 500$, the *RMI* of the modified EWMA control chart was 3.309499, while the *RMI* of the standard one was 2.134945.

Table 8 Comparison of the ARL of explicit formula of MA(3) process on EWMA and modified EWMA charts for $\lambda = 0.1$, $k = 0.15$, $\mu = 4$, $\theta_1 = -0.5$, $\theta_2 = -0.8$, and $\theta_3 = -0.6$, given $ARL_0 = 370$ and 500.

Shift size	$ARL_0 = 370$		$ARL_0 = 500$	
	EWMA ($h = 0.171, l = 2.54$)	**Mo EWMA ($a = 0.25, b = 1.728$)**	**EWMA ($h = 0.14, l = 1.0$)**	**Mo EWMA ($a = 0.41, b = 1.698$)**
0.00	370.130496	370.156260	500.520605	500.172701
0.01	349.661956	348.689900	471.131420	464.583020
0.02	330.721169	328.884699	444.014850	432.187541
0.05	281.763019	277.968305	374.305917	350.993818
0.10	220.341569	214.699103	287.742727	254.856364
0.20	144.247548	137.368071	182.237028	146.295192
0.50	59.906983	52.956080	68.431005	44.446754
1.00	29.523770	21.470957	27.399314	14.410122
2.00	26.580961	9.107241	12.165412	5.245699
RMI	**6.450327**	**0.000000**	**19.301145**	**0.000000**

The overall findings show that the modified EWMA control chart was better than the standard procedure for detecting a change in process mean.

Application of the control charts

Real observations were employed to determine the ARL by explicit formula of the EWMA and the modified EWMA control charts for Dow Jones composite average data. It is a stationary time series by looking at the autocorrelation function (ACF) and partial autocorrelation (PACF). The data were analysed and fitted to the 3rd-order of MA model with the white noise was tested to be significant to the exponential distribution. The performance of the modified EWMA control chart is presented through the ARL in **Table 9** and **Figure 2**.

Table 9 summarizes the ARL obtained by the explicit formulas of the EWMA and modified EWMA control charts for data comprising daily index of Dow Jones composite average from 28th December 2020 to 24th February 2021. These 40 observations were determined to be a 3rd-order moving average process with mean 10,235.937 and coefficients –0.930, –0.730, and -0.535 for the 1st-order, 2nd-order, and 3rd-order, respectively. The white noise was checked to ensure it was exponentially distributed and provided a mean of 52.4686.

The results indicate that the performance of the modified EWMA control chart is much more sensitive than the EWMA control chart, with changes in the mean for all magnitudes of shift size and zero of *RMI*.

Table 9 Comparison of the ARL of explicit formula of MA(3) process on EWMA and modified EWMA charts for $\beta_0 = 52.4686$, $\lambda = 0.1$, $k = 0.1$, $\mu = 10235.937$, $\theta_1 = -0.930$, $\theta_2 = -0.730$, and $\theta_3 = -0.535$, given $ARL_0 = 370$ and 500.

Shift size	$ARL_0 = 370$		$ARL_0 = 500$	
	EWMA (h = 0.124, l = 0.1685)	**Mo EWMA (a = 0.0299, b = 0.0301571)**	**EWMA (h = 0.1323, l = 0.19267)**	**Mo EWMA (a = 0.0302, b = 0.030551)**
0.00	370.032865	370.121701	500.095318	500.088549
0.50	331.491012	327.915026	447.982755	443.021078
1.00	297.559526	291.192686	402.102549	393.368944
5.00	134.032116	121.535731	180.966505	163.976307
10.0	57.072871	48.083607	76.872788	64.661802
20.0	15.016672	11.601814	19.972899	15.334753
30.0	5.822454	4.427027	7.529422	5.633704
40.0	3.062556	2.414285	3.793217	2.912265
50.0	2.030258	1.693714	2.395467	1.937976
100.0	1.115199	1.080001	1.156116	1.108170
RMI	**4.652947**	**0.000000**	**6.346097**	**0.000000**

In addition, the capability of the control schemes to detect the change in process mean is presented in **Figures 1** and **2**.

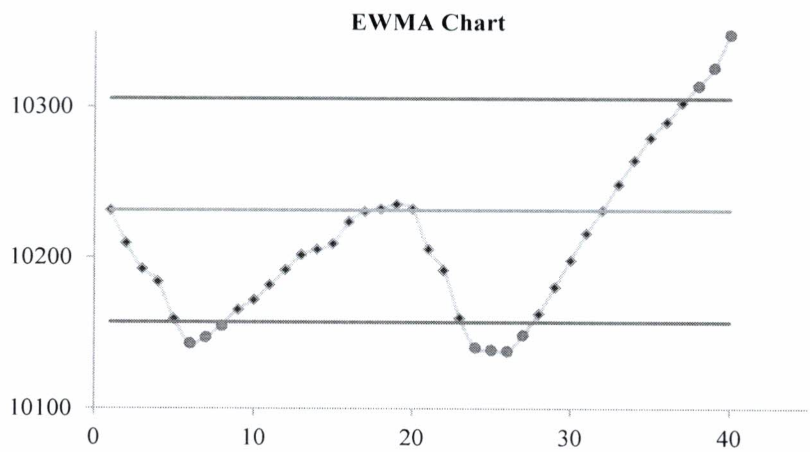

Figure 1 Detection of the changes in process mean by the EWMA control chart.

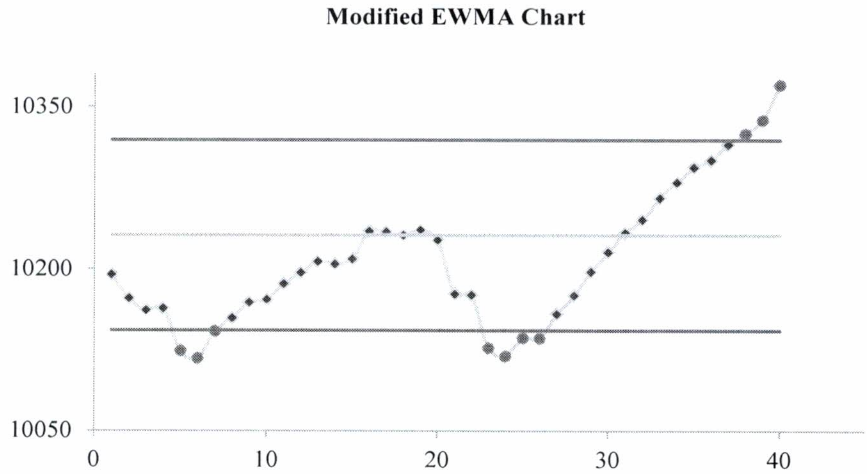

Figure 2 Detection of the changes in process mean by the modified EWMA control chart.

From **Figure 1**, we can see that the EWMA control chart detects lower shifts from the 6th to the 8th, and the 24th to the 27th, observations, and upper shifts from the 38th to the 40th observations. On the other hand, the modified EWMA control chart detects upper shifts from the 5th to the 7th, and the 23rd to the 26th, observations, and upper shifts from the 38th to the 40th observations (**Figure 2**).

Conclusions

An explicit formula is a good alternative to evaluate ARL, which is an effective measurement of a control chart. It yields the exact value, is easy to derive, and saves a great deal on computational time. To monitor and detect small or large changes in the autocorrelated observations of a general-order MA process with exponential white noise, the modified EWMA chart is recommended for taking proactive action before a process enters an out-of-control state. The findings of the application revealed that a smoothing parameter value of 0.1 and constant k value of 0.1 supported the modified EWMA procedure better than the EWMA control chart for all cases. Therefore, determination of the suitable value of smoothing parameter and constant k of the control chart should not be disregarded. However, the constant $k = -\lambda / 2$ is presented to be the minimize variance of the modified EWMA control statistic. Setting value to be the negative is the limitation of this study.

Suggestions for further study could be extended to Gaussian, Log-normal, or other distribution. Moreover, the Monte Carlo simulation, the Martingale and Markov chain approach, could be considered to evaluation of the ARL for normally independent or serially correlated observations.

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